Function

EXERCISES

ELEMENTRY

Q.1 (1)

The domain of function $\log_e\{x-[x]\}$ is R, because [x] is a greatest integer whose value is equal to or less than zero.

Q.2 (4)

$-1 \le \log_3 x \le 1$; $3^{-1} \le x \le 3 \Rightarrow \frac{1}{3} \le x \le 3$

$\therefore$ Domain of function $= \left[\frac{1}{3}, 3\right]$.

Q.3 (3)

(i) $x \le 2$

(ii) $\sqrt{9-x^2} > 0 \Rightarrow |x| < 3$

or $-3 < x < 3$

Hence domain is $(-3, 2]$.

Q.4 (3)

The function $f(x) = \sqrt{\log(x^2-6x+6)}$ is defined when $\log(x^2-6x+6) \ge 0$

$\Rightarrow x^2 - 6x + 6 \ge 1 \Rightarrow (x-5)(x-1) \ge 0$

This inequality holds if $x \le 1$ or $x \ge 5$. Hence, the domain of the function will be $(-\infty, 1] \cup [5, \infty)$.

Q.5 (3) $-1 \le 1 + 3x + 2x^2 \le 1$

Case I : $2x^2 + 3x + 1 \ge -1$; $2x^2 + 3x + 2 \ge 0$

$x = \frac{-3 \pm \sqrt{9-16}}{6} = \frac{-3 \pm i\sqrt{7}}{6}$ (imaginary).

Case II : $2x^2 + 3x + 1 \le 1$

$\Rightarrow 2x^2 + 3x \le 0 \Rightarrow 2x\left(x + \frac{3}{2}\right) \le 0$

$\Rightarrow \frac{-3}{2} \le x \le 0 \Rightarrow x \in \left[-\frac{3}{2}, 0\right]$

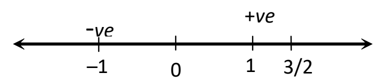
In case I, we get imaginary value hence, rejected

$\therefore$ Domain of function $= \left[\frac{-3}{2}, 0\right]$.

Q.6 (4)

$f(x) = e^{\sqrt{5x-3-2x^2}}$

$\Rightarrow 5x - 3 - 2x^2 \ge 0$ or $(x-1)\left(x - \frac{3}{2}\right) \ge 0$



$\therefore$ $D \in [1, 3/2]$

Q.7 (2)

To define f(x), $9 - x^2 > 3 \Rightarrow -3 < x < 3$ .....(i)

$-1 \le (x-3) \le 1 \Rightarrow 2 \le x \le 4$ .....(ii)

From (i) and (ii), $2 \le x < 3$ i.e., [2, 3).

Q.8 (3)

$f(x) = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} \Rightarrow$ Range $= (1, 7/3]$.

Q.9 (3)

$\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$ holds x lying in [0,1].

Q.10 (3)

$\frac{x^2 + 14x + 9}{x^2 + 2x + 3} = y$

$\Rightarrow x^2 + 14x + 9 = x^2y + 2xy + 3y$

$\Rightarrow x^2(y-1) + 2x(y-7) + (3y-9) = 0$

Since x is real, $\therefore 4(y-7)^2 - 4(3y-9)(y-1) > 0$

$\Rightarrow 4(y^2 + 49 - 14y) - 4(3y^2 + 9 - 12y) > 0$

$\Rightarrow 4y^2 + 196 - 56y - 12y^2 - 36 + 48y > 0$

$\Rightarrow 8y^2 + 8y - 160 < 0 \Rightarrow y^2 + y - 20 < 0$

$\Rightarrow (y+5)(y-4) < 0$; $\therefore$ y lies between – 5 and 4.

Q.11 (1)

$y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right] \Rightarrow -1 \le \log_3\left(\frac{x}{3}\right) \le 1$

$\Rightarrow \frac{1}{3} \le \frac{x}{3} \le 3 \Rightarrow 1 \le x \le 9 \Rightarrow x \in [1, 9]$.

Q.12 (4)

Here $x+3>0$ and $x^2+3x+2\neq 0$

$\therefore$ $x>-3$ and $(x+1)(x+2)\neq 0$, i.e. $x\neq -1,-2$

$\therefore$ Domain $=(-3,\infty)-\{-1,-2\}$.

Q.13 (2)

$x^2-6x+7=(x-3)^2-2$

Obviously, minimum value is – 2 and maximum ∞. Hence range of function is [–2, ∞].

Q.14 (3)

$f(x)=\log(\sqrt{x-4}+\sqrt{6-x})$

$\Rightarrow x-4\geq 0$ and $6-x\geq 0\Rightarrow x\geq 4$ and $x\leq 6$

$\therefore$ Domain of f(x) = [4, 6]

Q.15 (2)

The quantity under root is positive, when

$-1-\sqrt{3}\leq x\leq -1+\sqrt{3}$.

Q.16 (2)

Obviously, here $|x|>2$ and $x\neq 1$

i.e., $x\in(-\infty,-2)\cup(2,\infty)$.

Q.17 (2)

$\log\left\{\frac{5x-x^2}{6}\right\}\geq 0\Rightarrow\frac{5x-x^2}{6}\geq 1$

or $x^2-5x+6\leq 0$ or $(x-2)(x-3)$

Hence $2\leq x\leq 3$.

Q.18 (2)

We have $f(x)=(x-1)(x-2)(x-3)$

and $f(1)=f(2)=f(3)=0$

$\Rightarrow$ $f(x)$ is not one-one.

For each $y\in R$, there exists $x\in R$ such that $f(x)=y$. Therefore f is onto.

Hence $f:R\rightarrow R$ is onto but not one-one.

Q.19 (4)

$f(-1)=f(1)=1$;

$\therefore$ function is many-one function. Obviously, f is not onto so f is neither one-one nor onto.

Q.20 (1)

Let $x,y\in N$ such that $f(x)=f(y)$

Then $f(x)=f(y)\Rightarrow x^2+x+1=y^2+y+1$

$\Rightarrow(x-y)(x+y+1)=0\Rightarrow x=y$

or $x=(-y-1)\notin N$

$\therefore$ f is one-one.

Again, since for each $y\in N$, there exist $x\in N$

$\therefore$ f is onto.

Q.21 (3)

Let f(x) = f(y) $\Rightarrow\frac{x^2-4}{x^2+4}=\frac{y^2-4}{y^2+4}$

$\Rightarrow\frac{x^2-4}{x^2+4}-1=\frac{y^2-4}{y^2+4}-1\Rightarrow x^2+4=y^2+4$

$\Rightarrow x=\pm y$, $\therefore f(x)$ is many-one.

Now for each $y\in(-1,1)$, there does not exist $x\in X$ such that $f(x)=y$. Hence f is into.

Q.22 (2)

$f'(x)=\frac{1}{(1+x)^2}>0,\forall x\in[0,\infty)$ and range $\in[0,1)$

$\Rightarrow$ function is one-one but not onto.

Q.23 (4)

We have $f(x)=x+\sqrt{x^2}=x+|x|$

Clearly f is not one-one as $f(-1)=f(-2)=0$ but $-1\neq 2$.

Also f is not onto as $f(x)\geq 0,\forall x\in R$,

Also, range of $f=(0,\infty)\subset R$.

Q.24 (1)

$f[f(x)]=\frac{f(x)-3}{f(x)+1}$

$$=\frac{\left(\frac{x-3}{x+1}\right)-3}{\left(\frac{x-3}{x+1}\right)+1}=\frac{x-3-3x-3}{x-3+x+1}=\frac{3+x}{1-x}$$

Now $f[f(f(x))]=f\left(\frac{3+x}{1-x}\right)$

$$=\frac{\left(\frac{3+x}{1-x}\right)-3}{\left(\frac{3+x}{1-x}\right)+1}=\frac{3+x-3+3x}{3+x+1-x}=x.$$

Q.25 (2)

$f(2x)=2(2x)+|2x|=4x+2|x|$,

$f(-x)=-2x+|-x|=-2x+|x|$,

$f(x) = 2x + |x| \Rightarrow f(2x) + f(-x) - f(x)$

$= 4x + 2|x| + |x| - 2x - 2x - |x|$

Q.26 (4) Here $f(2) = \frac{5}{4}$

Hence $(fof)(2) = f(f(2)) = f\left(\frac{5}{4}\right) = \dfrac{2\times\frac{5}{4}+1}{3\times\frac{5}{4}-2} = 2$.

Q.27 (2)

$(gof)(x) = |\sin x|$ and $f(x) = \sin^2 x$

$\Rightarrow g(\sin^2 x) = |\sin x|$; $\therefore g(x) = \sqrt{x}$.

Q.28 (2)

$f(x) = \log(x + \sqrt{x^2+1})$

and $f(-x) = -\log(x + \sqrt{x^2+1}) = -f(x)$

f(x) is odd function.

Q.29 (4)

Given $f : (2,3) \to (0,1)$ and $f(x) = x - [x]$

$\therefore f(x) = y = x - 2 \Rightarrow x = y + 2 = f^{-1}(y) \Rightarrow f^{-1}(x) = x + 2$

Q.30 (1)

$y = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} \Rightarrow x = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 **(1)**

$f(x) = \sin^{-1}(|x-1| - 2)$

For domain $-1 \le |x-1| - 2 \le 1$

$\Rightarrow 1 \le |x-1| \le 3 \Rightarrow x - 1 \in [-3, -1] \cup [1, 3]$

$\Rightarrow x \in [-2, 0] \cup [2, 4]$

Q.2 **(4)**

For domain $-\log_{0.3}(x-1) \ge 0$ and $x^2 + 2x + 8 > 0$

$\Rightarrow \log_{0.3}(x-1) \le 0$ and

$\Rightarrow (x+1)^2 + 7 > 0$

$\Rightarrow (x-1) \ge 1$ and

$\Rightarrow x \in R$

$\Rightarrow x \ge 2$

Taking intersection $x \in [2, \infty)$

Q.3 **(3)**

$f(x) = \cot^{-1}\sqrt{x(x+3)} + \cos^{-1}\sqrt{x^2+3x+1}$

for domain $x(x+3) \ge 0$ and $0 \le x^2 + 3x + 1 \le 1$

$\Rightarrow x \in (-\infty, -3] \cup [0, \infty)$ and $x^2 + 3x + 1 \ge 0$ and $x^2 + 3x \le 0 \Rightarrow x \in [-3, 0]$

Taking intersection

$x \in \{-3, 0\}$

Q.4 **(4)**

$f(x) = \log_{1/2}\left(-\log_2\left(1 + \frac{1}{\sqrt[4]{x}}\right) - 1\right)$

$\Rightarrow -\log_2\left(1 + \frac{1}{x^{1/4}}\right) - 1 > 0$

$\Rightarrow -\log_2\left(1 + \frac{1}{x^{1/4}}\right) > 1$; $x > 0$

$\Rightarrow \log_2\left(1 + \frac{1}{x^{1/4}}\right) < -1$

$\Rightarrow 1 + \frac{1}{x^{1/4}} < \frac{1}{2}$

$\Rightarrow \frac{1}{x^{1/4}} < -\frac{1}{2}$

$\Rightarrow x \in \phi$

Q.5 **(2)**

$q^2 - 4pr = 0, p > 0$

$f(x) = \log(px^3 + (p+q)x^2 + (q+r)x + r)$

$\Rightarrow px^3 + (p+q)x^2 + (q+r)x + r > 0$

$\Rightarrow (px^3 + px^2) + (qx^2 + qx) + (rx + r) > 0$

$\Rightarrow px^2(x+1) + qx(x+1) + r(x+1) > 0$

$\Rightarrow (x+1)(px^2 + qx + r) > 0$

$\Rightarrow D = q^2 - 4pr$

Means it is perfect square D = 0 (given)

$x = -\frac{b}{2a}$

$(x+1)\left(x + \frac{q}{2p}\right)^2 > 0 \Rightarrow x + 1 > 0 \Rightarrow x > -1$ $\left(x \ne \frac{-q}{2p}\right)$

$\Rightarrow x \in (-1, \infty) - \left\{-\frac{q}{2p}\right\}$

$\therefore x \in R - \left[(-\infty, -1] \cup \left\{-\frac{q}{2p}\right\}\right]$

Q.6 **(1)**

$f(x) = \sqrt{-\log_{\frac{x+4}{2}}\left(\log_2\frac{2x-1}{3+x}\right)}$

Case-1 : $\frac{x+4}{2} > 1 \Rightarrow x > -2$

$$-\log_{\frac{x+4}{2}} \log_2\left(\frac{2x-1}{x+3}\right) \ge 0$$

$$\Rightarrow \log_{\frac{x+4}{2}} \log_2\left(\frac{2x-1}{x+3}\right) \le 0$$

$$\Rightarrow \log_2\left(\frac{2x-1}{x+3}\right) \le 1 \Rightarrow \frac{2x-1}{x+3} \le 2$$

$$\Rightarrow \frac{7}{x+3} \ge 0 \Rightarrow x > -3$$

$$\& \ \frac{2x-1}{x+3} > 0$$

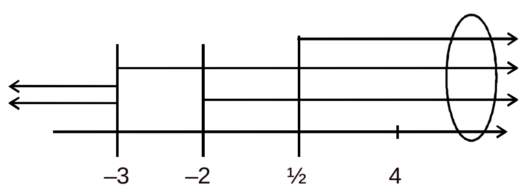
$$\Rightarrow x > \frac{1}{2}; x < -3$$

$$\& \ \log_2\left(\frac{2x-1}{x+3}\right) > 0$$

$$\Rightarrow \frac{2x-1}{x+3} > 1$$

$$\Rightarrow \frac{x-4}{x+3} > 0$$

$\Rightarrow$ x > 4 ; x < –3



$\therefore$ $x \in (4, \infty)$ ....(i)

Case-2 : $0 < \frac{x+4}{2} < 1 \Rightarrow -4 < x < -2$

$$-\log_{\frac{x+4}{2}} \log_2 \frac{2x-1}{x+3} \ge 0$$

$$\Rightarrow \log_{\frac{x+4}{2}} \log_2 \frac{2x-1}{x+3} \le 0 \Rightarrow \log_2 \frac{2x-1}{x+3} \ge 1$$

$$\Rightarrow \frac{7}{x+3} \le 0 \Rightarrow x < -3$$

$$\& \ \frac{2x-1}{x+3} > 0 \Rightarrow x > \frac{1}{2}; x < -3$$

$$\& \ \log_2\left(\frac{2x-1}{x+3}\right) > 0 \Rightarrow \frac{x-4}{x+3} > 0$$

$\Rightarrow$ x > 4 ; x < –3 $\therefore$ $x \in (-4, -3)$ ...(2)

(1) $\cup$ (2) $\Rightarrow$ $x \in (-4, -3) \cup (4, \infty)$

Q.7 **(1)**

$$\sqrt{\log_{1/3} \log_4([x]^2 - 5)}$$

$\Rightarrow \log_{1/3} \log_4([x]^2 - 5) \ge 0 \Rightarrow 0 < \log_4([x]^2 - 5) \le 1$

$\Rightarrow 1 < [x]^2 - 5 \le 4 \Rightarrow 6 < [x]^2 \le 9$

[x] always gives integer value so square of GTF will also give Integer value. In between 6 and 9 are only perfect square value possible.

$[x]^2 = 9 \Rightarrow [x] = 3$ $[x] = -3$

$3 \le x < 4$ $-3 \le x < -2$

$\therefore$ $x \in [-3, -2) \cup [3, 4)$

Q.8 **(1)**

$f(x) = \log_e(3x^2 - 4x + 5)$

$$3x^2 - 4x + 5 \ge \frac{11}{3}$$

$$\Rightarrow \ell n(3x^2 - 4x + 5) \ge \ell n \frac{11}{3}$$

[$\because$ ℓn is an increasing function]

$\therefore$ Range is $\left[\ell n \frac{11}{3}, \infty\right)$

Q.9 **(2)**

$f(x) = 4^x + 2^x + 1$

Let $2^x = t > 0, \forall x \in R$

$\therefore$ $f(x) = g(t) = t^2 + t + 1, t > 0$

$$g(t) = \left(t + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\left(t + \frac{1}{2}\right) > \frac{1}{2} \Rightarrow \left(t + \frac{1}{2}\right)^2 > \frac{1}{4}$$

$$\Rightarrow \left(t + \frac{1}{2}\right)^2 + \frac{3}{4} > 1$$

Range is $(1, \infty)$

Q.10 **(2)**

$f(x) = \log_{\sqrt{5}}\left(\sqrt{2}(\sin x - \cos x) + 3\right)$

we know that

$-\sqrt{2} \le \sin x - \cos x \le \sqrt{2}, \forall x \in R$

[since $-\sqrt{a^2 + b^2} \le a \sin x + b \cos x \le \sqrt{a^2 + b^2}$]

$\Rightarrow -2 \le \sqrt{2}(\sin x - \cos x) \le 2$

$\Rightarrow 1 \le \sqrt{2}(\sin x - \cos x) + 3 \le 5$

$\Rightarrow 0 \le \log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + 3) \le 2$

Hence range is [0, 2]

Q.11 **(4)**

$f(x) = \log_{\sqrt{2}}(2 - \log_2(16 \sin^2 x + 1))$

$\Rightarrow 0 \le \sin^2 x \le 1$ $\Rightarrow 0 \le 16 \sin^2 x \le 16$

$\Rightarrow 1 \le 16 \sin^2 x + 1 \le 17$

$\Rightarrow\ 0 \le \log_2 (16 \sin^2 x + 1) \le \log_2 17$

$\Rightarrow\ -\log_2 17 \le -\log_2 (16 \sin^2 x + 1) \le 0$

$\Rightarrow\ 2 - \log_2 17 \le 2 - \log_2 (16 \sin^2 x + 1) \le 2$

$\Rightarrow\ \log_{\sqrt{2}} (2 - \log_2 17) \le \log_{\sqrt{2}} M \le 2$

$\therefore\ -\infty < \log_{\sqrt{2}} M \le 2$

Q.12 **(3)**

$$f(x) = \begin{vmatrix} \cos\frac{x}{2} & 1 & 1 \\ 1 & \cos\frac{x}{2} & -\cos\frac{x}{2} \\ -\cos\frac{x}{2} & 1 & -1 \end{vmatrix} = 2 + 2\cos^2\frac{x}{2}$$

$0 \le \cos^2\frac{x}{2} \le 1 \Rightarrow 0 \le 2\cos^2\frac{x}{2} \le 2$

$\Rightarrow\ 2 \le 2 + 2\cos^2\frac{x}{2} \le 4 \quad \therefore\ 2 \le y \le 4$

Q.13 **(3)**

$$f(x) = \frac{\sin^2 x + 4\sin x + 5}{2\sin^2 x + 8\sin x + 8}$$

$$f(x) = \frac{1}{2}\left[\frac{2\sin^2 x + 8\sin x + 10}{2\sin^2 x + 8\sin x + 8}\right]$$

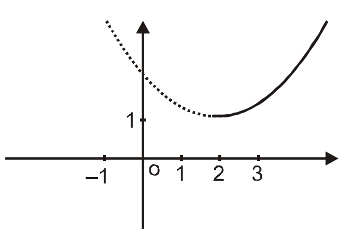
$$= \frac{1}{2}\left[1 + \frac{2}{2\sin^2 x + 8\sin x + 8}\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{(\sin x + 2)^2}\right]$$

$$f(x)|_{max.} = \frac{1}{2}[1 + 1] = 1$$

$$f(x)|_{min.} = \frac{1}{2}\left[1 + \frac{1}{9}\right] = \frac{5}{9} \quad \therefore \text{Range} \in \left[\frac{5}{9}, 1\right]$$

Q.14 **(2)**



$f : [2, \infty) \to Y$

$f(x) = x^2 - 4x + 5$

$f(x) = (x - 2)^2 + 1$

For given domain by graph range is $[1, \infty)$

For function to be onto codomain $y = [1, \infty)$

Q.15 **(4)**

$$f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}, \text{ Domain } x \in R$$

$f'(x)$

$$= \frac{(4x-1)(7x^2+2x+10) - (14x+2)(2x^2-x+5)}{(7x^2+2x+10)^2}$$

$$f'(x) = \frac{11x^2 - 30x - 20}{(7x^2+2x+10)^2} > 0 \Rightarrow x \in (-\infty, 0) \cup \left(\frac{30}{11}, \infty\right)$$

$$f'(x) < 0 \Rightarrow x \in \left(0, \frac{30}{11}\right)$$

$$f'(x) = 0 \Rightarrow x = 0, \frac{30}{11}$$

Function is increasing and decreasing in different intervals, so non monotonic

$\therefore$ Many one function.

<u>Onto / Into</u>

$$f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}$$

$2x^2 - x + 5 > 0, \forall x \in R$ and $7x^2 + 2x + 10 > 0\ \forall x \in R$

$\because\ a = 2 > 0$ and

$\because\ a = 7$ and $D = 4 - 280 < 0$

$D = 1 - 40 = -39 < 0$

$\therefore\ f(x) > 0\ \forall x \in R$

Also f(x) never tends to $\pm\infty$ as $7x^2 + 2x + 10$ has no real roots, Range $\ne$ Codomain so into function.

Q.16 **(1)**

$f(x) = x^3 + x^2 + 3x + \sin x, x \in R$

$f'(x) = 3x^2 + 2x + 3 + \cos x$

$\because\ 3x^2 + 2x + 3 \ge \frac{32}{12}$ as $a = 3 > 0$ and $D < 0$

$-1 \le \cos x \le 1$

so $f'(x) > 0\ \forall x \in R$

$$\lim_{x\to\infty} f(x) = +\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

Hence f(x) is one-one and onto function (as f(x) is continuous function)

Q.17 **(1)**

$$f(x) = \frac{4a-7}{3}x^3 + (a-3)x^2 + x + 5$$

Case – 1 : a = $\frac{7}{4}$

f(x) = $-\frac{5}{4}x^2 + x + 5$ which can't be one-one

Case – 2 : a ≠ $\frac{7}{4}$

$f'(x) = (4a - 7)x^2 + 2(a - 3)x + 1$

$D \le 0$

$\Rightarrow\ 4(a-3)^2 - 4(4a-7) \le 0$

$\Rightarrow\ a^2 - 6a + 9 - 4a + 7 \le 0$

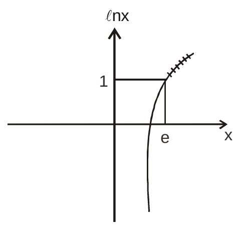
$\Rightarrow\ a^2 - 10a + 16 \le 0$

$\Rightarrow\ (a-8)(a-2) \le 0 \qquad \therefore\ 2 \le a \le 8$

Q.18 (**3**)

$f : (e, \infty) \to R$

$f(x) = \ell n(\ell n(\ell n\, x))$

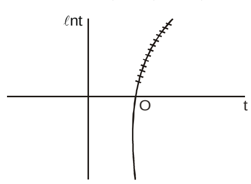


$e < x < \infty$

$1 < \ell nx < \infty$

$0 < \ell n(\ell n\, x) < \infty$

$-\infty < \ell n(\ell n(\ell n\, x)) < \infty$

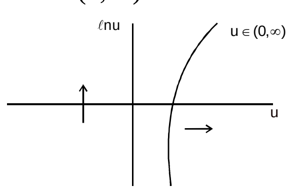


Range = Co-domain onto

Let $\ell n\, t = u$

Let $\ell nx = t$; $t \in (1, \infty)$

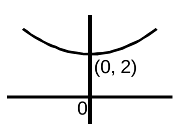
$\ell n\, t \in (0, \infty)$



one-one

Hence, one-one onto

Q.19 (**4**)



$f : R \to R$; $f(x) = 6^x + 6^{|x|}$

$x \ge 0 \quad f(x) = 2.6^x$

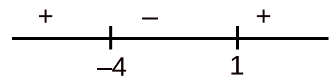
$x < 0 \quad f(x) = 6^x + 6^{-x} = 6^x + \frac{1}{6^x} \ge 2$

Many-one into

Q.20 (**3**)

$f : R \to R$; $f(x) = \frac{x^2 - 4}{x^2 + 1} \Rightarrow f(-x) = \frac{x^2 - 4}{x^2 + 1} = f(x)$

f(x) is even that's why many-one.



$y = \frac{x^2 - 4}{x^2 + 1} \Rightarrow yx^2 + y = x^2 - 4$

$\Rightarrow\ x^2 = \frac{y+4}{1-y} \ge 0$

$\Rightarrow\ \frac{y+4}{y-1} \le 0$

$\therefore\ y \in [-4, 1)$

Range ≠ Co-Domain ⇒ into

Q.21 (**1**)

$\frac{y}{1} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

By compnendo and dividendo

$\frac{1+y}{1-y} = \frac{2e^x}{2} \Rightarrow \quad 2x = \ell n\left(\frac{1+y}{1-y}\right) \quad \Rightarrow \quad x = \frac{1}{2}\ell n \left(\frac{1+y}{1-y}\right)$

$\therefore\ f^{-1}(x) = \frac{1}{2}\ell n\left(\frac{1+x}{1-x}\right)$

Q.22 (**3**)

$f(x) = x - \left[\frac{x}{2}\right]$

for the given domain (2, 4)

$\left[\frac{x}{2}\right]$ will be equal to 1; so $y = f(x) = x - 1$

$\Rightarrow\ x = y + 1 \qquad \Rightarrow\ f^{-1}(x) = x + 1$

Q.23 (**3**)

Domain of f(g(x))

Range of g(x) ≡ Domain of f(x)

$\Rightarrow\ -5 \le |2x + 5| \le 7$

$\Rightarrow\ 0 \le |2x + 5| \le 7$

$\Rightarrow\ -7 \le 2x + 5 \le 7$

$\Rightarrow\ -12 \le 2x \le 2$

$\Rightarrow\ -6 \le x \le 1$

Q.24 (**1**)

$f(x) = \frac{ax + b}{cx + d}$

$$fof(x) = \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d}$$

$$fof(x) = \frac{a^2x+ab+bcx+bd}{acx+bc+cdx+d^2}$$

$$fof(x) = \frac{(a^2+bc)x+(ab+bd)}{(ac+cd)x+(bc+d^2)} = x$$

on comparing coefficient of both side $(a^2 + bc)x + (ab + bd) = (ac + cd)x^2 + (bc + d^2)x$

$a^2 + bc = bc + d^2 \Rightarrow a = d$ or $a = -d$

and $ab + bd = 0 \Rightarrow b = 0$ or $a = -d$

and $ac + cd = 0 \Rightarrow c = 0$ or $a = -d$

which can be simultaneously true for $a = -d$

Q.25 (1)

$f(g(x_1)) = f(g(x_2))$

$\Rightarrow g(x_1) = g(x_2)$

as f is one - one function

$\Rightarrow x_1 = x_2$

as g is one - one function

hence $f(g(x_1)) = f(g(x_2))$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ f(g(x)) is one - one function

Q.26 (3)

$f(g(x)) = \cot^{-1}(2x - x^2)$

$-\infty < 2x - x^2 \le 1$

But domain of f(x) is R^+

$0 < 2x - x^2 \le 1$

$\Rightarrow \frac{\pi}{2} > \cot^{-1}(2x - x^2) \ge \frac{\pi}{4}$

Range $\in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

Q.27 (2)

$g(x) = 1 + x - [x]$ $f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

$\Rightarrow$ f(x) = sgnx

$f[g(x)] = f(1+ x - [x]) = \text{sgn}(1+ x - [x])$

$= \text{sgn}(\underset{\substack{\downarrow \\ \text{positive}}}{1+\{x\}}) = 1$

Q.28 (1)

$f : [0, 1] \to [1, 2]$ $g : [1, 2] \to [0, 1]$

$f(x) = 1 + x$

$g(x) = 2 - x$

$gof(x) = g[f(x)] = g(1 + x) = 2 - (1 + x) = 1 - x$

Linear polynomial thats why one-one onto.

Q.29 (4)

$f(x) = \frac{x+|x|}{2} = \begin{cases} x & ; x \ge 0; g(x) = x^2, x \ge 0 \\ 0 & ; x < 0; g(x) = x \ x < 0 \end{cases}$

$gof = g[f(x)] = \begin{cases} g(x) & ; x \ge 0 \\ x^2 & ; x \ge 0 \\ g(0) & ; x < 0 \\ 0 & ; x < 0 \end{cases}$

$\Rightarrow fog(x) = f[g(x)] = \begin{cases} f(x^2) & ; x \ge 0 \\ x^2 & ; x \ge 0 \\ f(x) & ; x < 0 \\ 0 & ; x < 0 \end{cases}$

$\Rightarrow fog(x) = gof(x)$.

Q.30 (3)

$f(x) = \sec(\sin x)$

Since sin x is a periodic function with fundamental period 2π. f(x) has a period 2π

for fundamental period

$f(x + \pi) = \sec(\sin(\pi + x)) = \sec(-\sin x) = \sec(\sin x) = f(x)$

$f\left(x+\frac{\pi}{2}\right) \ne f(x)$ hence fundamental period is π

Q.31 (4)

$f(x) = \sin\left(\sqrt{[a]}\,x\right)$

$\text{Period} = \frac{2\pi}{\sqrt{[a]}} = \pi$

$[a] = 4$

$\Rightarrow a \in [4, 5)$

Q.32 (1)

$f(x) = x + a - [x + b] + \sin \pi x + \cos 2\pi x + \sin(3\pi x) + \cos(4\pi x) + \dots\dots\dots + \sin(2n - 1)\pi + \cos(2px)$

$f(x) = \{x + b\} + a - b + \sin(\pi x) + \cos(2\pi x) + \sin(3\pi x) + \cos(4\pi x) + \dots. + \sin(2n - 1) + \cos(2n\pi x)$

Period of f(x) = L.C.M $\left(1, 2, \frac{2}{3}, \frac{2}{4}, \dots\dots\dots, \frac{2}{2n-1}, \frac{2}{2n}\right) = 2$

$\therefore$ period of f(x) = 2

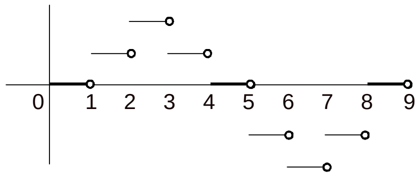
since $f(1 + x) \ne f(x)$, hence fundamental period is 2

Q.33 (3)

$f(x) = \sin\frac{\pi}{4}[x] + \cos\frac{\pi x}{2} + \cos\frac{\pi}{3}[x]$

If $0 \le x < 1$ then y = 0

$\frac{2\pi}{\left(\frac{\pi}{4}\right)} = 8$



If $1 \le x < 2$ then $y = \frac{1}{\sqrt{2}}$

If $2 \le x < 3$ then $y = 1$

If $3 \le x < 4$ then $y = \frac{1}{\sqrt{2}}$

If $4 \le x < 5$ then $y = 0$

Period 8/4/6 $\Rightarrow$ Lcm = 24

Q.34 **(3)**

$f(x) = x(2 - x)$; $f(x + 2) = f(x)$

$\Rightarrow$ period = 2

Q.35 **(2)**

$$f(x) = \log\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$f(-x) = \log\left(\frac{1-\sin x}{1+\sin x}\right) = -\log\left(\frac{1+\sin x}{1-\sin x}\right) = -f(x)$$

odd function

Q.36 **(2)**

$$f(x) = [x] + \frac{1}{2},\ x \notin I$$

$$f(-x) = [-x] + \frac{1}{2} = -[x] - 1 + \frac{1}{2} = -\left([x] + \frac{1}{2}\right) = -f(x)$$

odd function

Q.37 **(4)**

$$f(x) = \frac{xf(x^2)}{2 + \tan^2 x.f(x^2)}$$ given that $f(-x) = f(x)$ ....(1)

$$f(-x) = \frac{-xf(x^2)}{2 + \tan^2 x + f(x^2)} \Rightarrow f(-x) = -f(x)$$ ...(2)

When both conditions are there only one possibility is there when $f(x) = 0 \Rightarrow f(10) = 0$

Q.38 **(1)**

$$f(x) = \frac{ax - c}{cx - a} = y$$

$$f(y) = \frac{ay - c}{cy - a} = \frac{a\left(\frac{ax-c}{cx-a}\right) - c}{c\left(\frac{ax-c}{cx-a}\right) - a}$$

$$= \frac{a^2x - ac - c^2x + ac}{acx - c^2 - acx + a^2} = x$$

Q.39 **(4)**

$$f(x) = \cos\left[\frac{\pi^2}{2}\right]x + \sin\left[\frac{\pi^2}{2}\right]x\ ;\ \frac{\pi^2}{2} \cong 4.9$$

$f(x) = \cos 4x - \sin 5x$

$f(0) = 1$

$$f\left(\frac{\pi}{3}\right) = \cos\frac{4\pi}{3} - \sin\frac{5\pi}{3} = \frac{\sqrt{3}-1}{2} = \frac{1}{\sqrt{3}+1}$$

$$f\left(\frac{\pi}{2}\right) = 0$$

$f(\pi) = 1$

Q.40 **(4)**

$f(x) = |x - 1|$

$f(x^2) = |x^2 - 1|$ and $f^2(x) = |x - 1|^2$

$\Rightarrow f(x^2) \ne f^2(x)$

$f(x + y) = |x + y - 1|$ and $f(x) + f(y) = |x - 1| + |y - 1|$

$\Rightarrow f(x + y) \ne f(x) + f(y)$

$f(|x|) = ||x| - 1|$ and $|f(x)| = ||x - 1||$

Q.41 **(2)**

(1) $f(x) = \sin^{-1}x + \cos^{-1}x,\ x \in [-1, 1]$ and $g(x) = \frac{\pi}{2}$, $x \in R$

$f(x) = \frac{\pi}{2},\ x \in [-1, 1]$ and $g(x) = \frac{\pi}{2},\ x \in R$ Non-identical functions

(2) $f(x) = \tan^{-1}x + \cot^{-1}x$ and $g(x) = \frac{\pi}{2},\ x \in R$

$f(x) = \frac{\pi}{2},\ x \in R$ and $g(x) = \frac{\pi}{2},\ x \in R$ Identical functions

(3) $f(x) = \sec^{-1}x + \text{cosec}^{-1}x$ and $g(x) = \frac{\pi}{2},\ x \in R$

$f(x) = \frac{\pi}{2},\ |x| \in [1, \infty)$ and $g(x) = \frac{\pi}{2},\ x \in R$ Non-identical functions

Q.42 **(1)**

$$\left[\frac{1}{2} + \frac{1999}{2000}\right] = [0.5+0.995] = 1\ ⋮$$

(first thousand terms

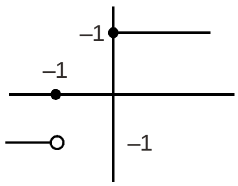
$$\left[\frac{1}{2} + \frac{1000}{2000}\right] = [0.5 + 0.5] = 1$$

will be equal to 0)

$(1 + 1 + ... + 1)_{1000 \text{ times}} = 1000$

Q.43 **(1)**

$f(x) = \text{sgn}\,[x + 1]$



$= 1$ if $[x + 1] > 0$

$\Rightarrow [x] > -1 \quad \therefore \quad x \ge 0$

$= 0$ if $[x + 1] = 0$

$\Rightarrow [x] = -1 \therefore -1 \le x < 0$

$= -1$ if $[x + 1] < 0$

$\Rightarrow [x] < -1 \quad \therefore \quad x < -1$

Q.44 **(2)**

$f(x) = 2\sin^2\theta + 4\cos(x + \theta)\sin x.\sin\theta + \cos(2x+2\theta)$

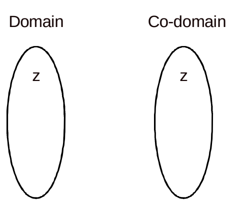
$f(x) = \cos 2x$

$\Rightarrow \quad f\left(\frac{\pi}{4} - x\right) = \cos\left(2\left(\frac{\pi}{4} - x\right)\right) = \sin 2x$

$\Rightarrow \quad f^2(x) + f^2\left(\frac{\pi}{4} - x\right) = 1$

Q.45 **(2)**

$z \to$ integer



$f(x) = ax^2 + bx + c$

$f(0) = c \in I$

$f(1) = a + b + c = I$

$f(1) - f(0) = a + b \in I$

Q.46 **(3)**

(A) $e^{(\ell n x)/2}$ and $\sqrt{x} \Rightarrow D_1 \in (0, \infty)$; $D_2 \in [0, \infty)$

Domain are not same so not identical

(B) $\tan^{-1}(\tan x)$ and $\cot^{-1}(\cot x)$

Domain are not same so non identical

(C) $\cos^2x + \sin^4x$ and $\sin^2x + \cos^4x$

$\Rightarrow x \in R$ & $x \in R$ Identical

Q.47 **(4)**

$f(x) = \cos(\ell n\, x)$

$f(x) . f(y) = \cos(\ell n\, x) . \cos(\ell n\, y)$

$f\left(\frac{x}{y}\right) + f(xy) = \cos(\ell n\, x - \ell n y) + \cos(\ell n\, x + \ell n\, y)$

$\therefore \quad f(x) . f(y) - \frac{1}{2}\left(f\left(\frac{x}{y}\right) + f(xy)\right) = \cos(\ell n\, x)\cos$

$(\ell n\, y) - \frac{1}{2}$ [2 $\cos(\ell n x)\cos(\ell n\, y) = 0$

Q.48 **(4)**

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

Replace $x + \frac{1}{x} = t$, where $|t| \ge 2$

$\therefore \quad f(t) = t^2 - 2, |t| \ge 2$

Q.49 **(3)**

$f(1) = 1 = 2 - 1$

$f(n + 1) = 2f(n) + 1$

$\therefore \quad f(2) = 2f(1) + 1 = 2.\,1 + 1 = 3 = 2^2 - 1$

$f(3) = 7 = 2^3 - 1$

$f(4) = 15 = 2^4 - 1$

Similarly $f(n) = 2^n - 1$

Q.50 **(2)**

Method 1 : (usual but lengthy)

$x^2 f(x) + f(1 - x) = 2x - x^4$ .....(1)

replace x by $(1 - x)$ in equation (1)

$(1 - x)^2 f(1 - x) + f(x) = 2(1 - x) - (1 - x)^4$ .....(2)

eliminate $f(1 - x)$ by equation (1) and (2)

we get

$f(x) = 1 - x^2$

Method 2 :

Since R.H.S. is polynomial of 4th degree and also by options consider $f(x) = ax^2 + bx + c$

$x^2 f(x) + f(1 - x) = 2x - x^4$

$\Rightarrow \quad x^2(ax^2 + bx + c) + a(1 - x)^2 + b(1 - x) + c = 2x - x^4$

by comparing coefficients

$a = -1$

$b = 0$

$c = 1$

$\therefore \quad f(x) = -x^2 + 1$

Q.51 **(3)**

$y = 2[x] + 3$ and $y = 3[x - 2]$

$2[x] + 3 = 3[x] - 6$

$\Rightarrow \quad [x] = 9 \Rightarrow x \in [9, 10)$

$\therefore \quad y = 21$

$\therefore \quad [x + y] = 30$

Q.52 **(4)**

$y = f(x)$

$f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \neq 0)$

$f\left(x+\frac{1}{x}\right) = \left(x+\frac{1}{x}\right)^2 - 2$

$\Rightarrow$ $f(x) = x^2 - 2$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 **(D)**

$$f(x) = \sqrt{\frac{1}{(|x|-1)\cos^{-1}(2x+1)\tan 3x}}$$

here $-1 \leq 2x + 1 < 1$ $\Rightarrow -2 \leq 2x < 0 \Rightarrow -1 \leq x < 0$

$\Rightarrow$ $x \in [-1, 0)$

But $x \neq -1$ as $|x| - 1 \neq 0$

$\therefore$ $x \in (-1, 0)$

for $x \in (-1, 0)$, $(|x| - 1)$ is –ve

$\therefore$ $\tan 3x < 0$

$0 > 3x > -\frac{\pi}{2}$ or $x \in \left(-\frac{\pi}{6}, 0\right)$

Domain : $\left(-\frac{\pi}{6}, 0\right) \cap (-1, 0) \equiv \left(-\frac{\pi}{6}, 0\right)$

Q.2 **(D)**

$$f(x) = \sin^{-1}\left(\frac{1+x^3}{2x^{3/2}}\right) + \sqrt{\sin(\sin x)} + \log_{(3\{x\}+1)}(x^2+1)$$

Domain : $3\{x\} + 1 \neq 1$ or 0

$\Rightarrow$ $x \notin I$

and $-1 \leq \frac{1+x^3}{2x^{3/2}} \leq 1$

$-2x^{3/2} \leq 1 + x^3 \leq 2x^{3/2}$

$1 + x^3 + 2x^{3/2} \geq 0$

$(1 + x^{3/2})^2 \geq 0$

$\Rightarrow$ $x \in R$

$1 + x^3 - 2x^{3/2} \leq 0$

or $(1 - x^{3/2})^2 \leq 0$

or $1 - x^{3/2} = 0$ or $x = 1$

Hence domain $x \in \phi$

Q.3 **(A)**

Domains of f(x) is $(-\infty, 0]$

Domains of f $(6\{x\}^2 - 5\{x\} + 1)$

$6\{x\}^2 - 5\{x\} + 1 \leq 0 \Rightarrow \frac{1}{3} \leq \{x\} \leq \frac{1}{2}$

$\Rightarrow n + \frac{1}{3} \leq x \leq n + \frac{1}{2}$; $n \in I$ $\Rightarrow \bigcup_{n\in I}\left[n+\frac{1}{3}, n+\frac{1}{2}\right]$

Q.4 **(D)**

$f(x) = \cot^{-1}(x^2 - 4x + 3)$,

Domain $x \in R$

range of $x^2 - 4x + 3$ is $[-1, \infty)$

$-1 \leq x^2 - 4x + 3 < \infty$

$\frac{3\pi}{4} \geq \cot^{-1}(x^2 - 4x + 3) > 0$

Range $y \in \left(0, \frac{3\pi}{4}\right]$

Q.5 **(D)**

$f(x) = (\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x)$

$= \frac{\pi^3}{8} - 3\sin^{-1}x\left(\frac{\pi}{2} - \cos^{-1}x\right)\frac{\pi}{2} = \frac{\pi^3}{8} - \frac{3\pi^2}{4}\sin^{-1}x + 3\frac{\pi}{2}(\sin^{-1}x)^2$

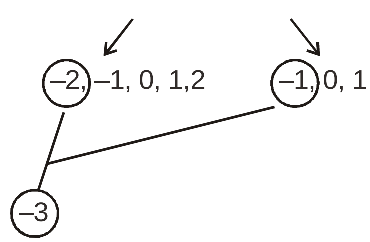
$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1}x)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{16}\right] - \frac{3\pi^3}{32} = \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1}x - \frac{\pi}{4}\right)^2$

maximum value of f(x) at $x = -1$

$f_{maximum} = \frac{\pi^3}{32} + \frac{3\pi}{2} \times \frac{9\pi^3}{16} = \frac{7\pi^3}{8}$

Q.6 **(D)**

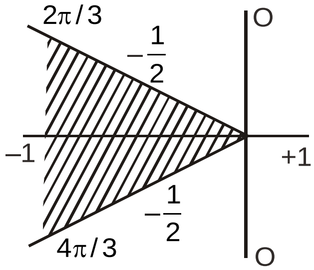
$[2\cos x] + [\sin x] = -3$



$-2 \leq 2\cos x < -1$

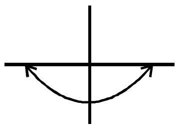
$-1 \leq \cos x < -\frac{1}{2}$

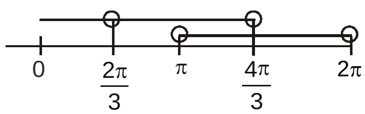
$x \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$



$-1 \le \sin x < 0$

$x \in (\pi, 2\pi)$





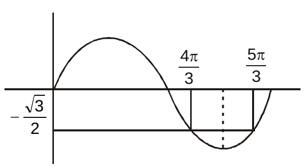
$\therefore\ \pi < x < \frac{4\pi}{3}$

$f(x) = \sin x + \sqrt{3}\cos x$

$= 2\left[\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right] = 2\sin\left(x + \frac{\pi}{3}\right) = 2\sin\theta$

$\pi + \frac{\pi}{3} < \theta < \frac{\pi}{3} + \frac{4\pi}{3}$

$\Rightarrow \frac{4\pi}{3} < \theta < \frac{5\pi}{3}$



$\Rightarrow\ -1 \le \sin\theta < -\frac{\sqrt{3}}{2}$

$\Rightarrow\ -2 \le 2\sin\theta < -\sqrt{3}$

$\therefore\ [-2, -\sqrt{3})$

Q.7 **(B)**

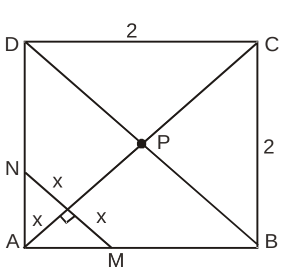
Area (ΔAMN)

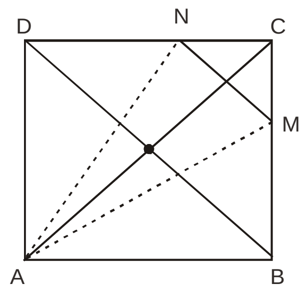
$= \frac{1}{2}(2x)\,x = x^2$

$(AP = \sqrt{2})$

$0 < x \le \sqrt{2}$

$0 < x^2 \le 2$





$\sqrt{2} \le x < 2\sqrt{2}$

$PC = 2\sqrt{2} - x$

$MN = 2(2\sqrt{2} - x)$

Area (ΔAMN)

$= \frac{1}{2}\,2(2\sqrt{2} - x)\,x$

$= 2\sqrt{2}\,x - x^2 = -(x^2 - 2\sqrt{2}\,x)$

$= -[(x - \sqrt{2})^2 - 2] = 2 - (x - \sqrt{2})^2$

$\Rightarrow\ x = \sqrt{2}, y = 2\ ;\ x = 2\sqrt{2}, y = 0\ \therefore\ y \in (0, 2]$

Q.8 **(C)**

$f(x) = \frac{x - [x]}{1 + x - [x]} = \frac{\{x\}}{1 + \{x\}} = 1 - \frac{1}{1 + \{x\}}$

$\because\ \{x\} \in [0, 1) \Rightarrow\ f(x) \in \left[0, \frac{1}{2}\right)$

Q.9 **(D)**

Here $(2 - \log_2(16\sin^2 x + 1) > 0$

$\Rightarrow\ 0 < 16\sin^2 x + 1 < 4$

$\Rightarrow\ 0 \le \sin^2 x < \frac{3}{16}$

$\Rightarrow\ 1 \le 16\sin^2 x + 1 \le 4$

$\Rightarrow\ 0 \le \log_2(16\sin^2 x + 1) < 2$

$\Rightarrow\ 2 \ge 2 - \log_2(16\sin^2 x + 1) > 0$

$\Rightarrow\ \log_{\sqrt{2}} 2 \ge \log_{\sqrt{2}}(2 - \log_2(16\sin^2 x + 1)) > -\infty$

$\Rightarrow\ 2 \ge y > -\infty$

Hence range is $y \in (-\infty, 2]$

Q.10 **(D)**

$f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}} = \begin{cases} \frac{e^x - e^{-x}}{2e^x} & ;\ x \ge 0 \\ 0 & ;\ x < 0 \end{cases}$

for $x \ge 0$

$y = \frac{e^x - e^{-x}}{2e^x} = \frac{e^{2x} - 1}{2e^{2x}}$

Let $t = e^x \Rightarrow y = \frac{t^2 - 1}{2t^2}$

$e^x \geq 1 \Rightarrow t \geq 1$

$\therefore \quad 2yt^2 = t^2 - 1$

$t^2 \geq 1 \Rightarrow t^2 = \frac{1}{1-2y} \geq 1 \Rightarrow 0 \leq y < \frac{1}{2}$

Q.11 **(C)**

$f(x) = 2[x] + \cos x$

$f(x) = \cos x \qquad x \in [0, 1)$

$= 2 + \cos x \qquad x \in [1, 2)$

$= 4 + \cos x \qquad x \in [2, 3)$

$= 6 + \cos x \qquad x \in [3, 4)$

for $x \in [0, 1) \qquad f'(x) = -$ ve

$x \in [1, 2) \qquad f'(x) = -$ ve

$x \in [2, 3) \qquad f'(x) = -$ ve

$x \in [3, 4) \qquad f'(x) = +$ ve

$\Rightarrow$ function is not one-one

if $x \in [0, 1)$ range : $[1, \cos 1)$

$x \in [1, 2)$ range : $[2 + \cos 1, 2 + \cos 2)$

not onto function

Q.12 **(D)**

$f(x) = px + \sin x$

$f'(x) = p + \cos x$

$p \neq 0$ for converging ranges of f(x) is $(-\infty, \infty)$

$f'(x) = P + \cos x > 0$ or $< 0 \quad \forall x \in R$

$P \in (-\infty, -1) \cup (1, \infty) \to f'(x)$ will not be zero.

Q.13 **(B)**

$f : S \to R^+$, $f(\Delta)$ = area of the Δ

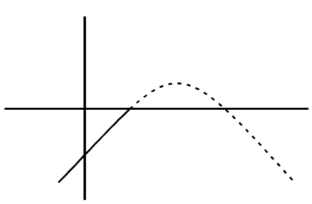
$S \to$ set of triangle ; $R^+ \to$ set of real values

for one base there are many triangle can possible. So many-one.

Q.14 **(D)**

$f : (-\infty, 1) \to [0, e^5]$; $f(x) = e^{-(x^2-3x+2)} = e^{-x^2+3x-2}$

$t(x) = -x^2 + 3x - 2$



Range $(-\infty, 0]$

$= -(x-2)(x-1)$

$-\infty < t(x) < 0$

$0 < e^{t(x)} < 1$

$f(x) = e^{t(x)}$

$f(x_1) = f(x_2)$

$e^{t(x_1)} = e^{t(x_2)} \qquad \Rightarrow t(x_1) = t(x_2)$

$x_1 = x_2 \Rightarrow t$ is one-one $\qquad \Rightarrow$ f is one-one function.

Q.15 **(A)**

$xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = f^{-1}(y) = \frac{3y-2}{y-1}$

Q.16 **(B)**

x = 10 satisfies

f (x) = x

Q.17 **(C)**

$f(x) = \cot^{-1}x \qquad R^+ \to \left(0, \frac{\pi}{2}\right)$

$g(x) = 2x - x^2 \qquad R \to R$

$f(g(x)) = \cot^{-1}(2x - x^2)$, where $x \in (0, 1]$

hence $f(g(x)) \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

Q.18 **(B)**

$f(x) = |x - 1| \qquad f : R^+ \to R$

$g(x) = e^x, \qquad g : [-1, \infty) \to R$

$fog(x) = f[g(x)] = |e^x - 1|$

$D : [-1, \infty)$

$R : [0, \infty)$

Q.19 **(A)**

$f\left(x + \frac{1}{3}\right) = \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] + [x + 1] - 3\left(x + \frac{1}{3}\right) + 15$

$= \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] + [x] - 3x + 15 = f(x)$

$\therefore$ fundamental period is 1/3

Q.20 **(D)**

$f(x) = \sin\sqrt{[a]}\,x \Rightarrow \frac{2\pi}{\sqrt{[a]}} = \pi$

$\Rightarrow 4 = [a] \quad \therefore a \in [4, 5)$

Q.21 **(B)**

$f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1 = \frac{x}{2}\left(\frac{e^x + 1}{e^x - 1}\right) + 1$

$f(-x) = -\frac{x}{2}\left(\frac{e^{-x} + 1}{e^{-x} - 1}\right) + 1 = \frac{x}{2}\left(\frac{e^x + 1}{e^x - 1}\right) = f(x)$

even function

Q.22 **(D)**

$$f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$$

$\because$ f(x) = f(–x)

$$\Rightarrow \frac{1 - a^x}{(-x)^n(1 + a^x)} = \frac{a^x - 1}{x^n(a^x + 1)} \Rightarrow (-x)^n = -x^n$$

$$n = -\frac{1}{3}$$

Q.23 **(C)**

$$g : [-2, 2] \to R ;\ g(x) = x^3 + \tan x + \left[\frac{x^2+1}{p}\right]$$

g(– x) = – g(x)

$$\Rightarrow \left[\frac{x^2+1}{p}\right] = 0$$

$$\therefore \quad 0 \le \frac{x^2+1}{p} < 1$$

$$\Rightarrow \quad x^2 + 1 = 5 \ \Rightarrow\ \frac{5}{p} < 1$$

$\Rightarrow$ p > 5

Q.24 **(D)**

(A) $\sqrt{1+\sin x} = \left|\sin\frac{x}{2} + \cos\frac{x}{2}\right|$

$\sin\frac{x}{2} + \cos\frac{x}{2}$

non-identical function

(B) $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$, $x \in [-1, 1]$ only

$2\tan^{-1}x$, $x \in R$

non-identical function

(C) $\sqrt{x^2} = |x|$, $x \in R$

$\left(\sqrt{x}\right)^2 = x$, $x \in R^+ \cup \{0\}$

non-identical function

(D) $\ell n x^3 + \ell n x^2 = 5\,\ell n x$, $x > 0$

$5\ell x$, $x > 0$

identical function

Q.25 **(C)**

[x] + 2 {– x} = 3x

Case–1 : $x \in I$; $x + 0 = 3x \Rightarrow 2x = 0 \Rightarrow x = 0$

Case–2 : $x \notin I$

$\Rightarrow [x] + 2(1 - \{x\}) = 3x \Rightarrow [x] + 2 - 2(x - [x]) = 3x$

$\Rightarrow [x] + 2 - 2x + 2[x] = 3x \Rightarrow 3[x] = 5x - 2$ ...(1)

& $x - \{x\} + 2 - 2\{x\} = 3x$

$\Rightarrow 2 - 3\{x\} = 2x \Rightarrow 0 \le \{x\} < 1$

$\Rightarrow 0 \le 3\{x\} < 3 \Rightarrow 0 \le 2 - 2x < 3$

$\Rightarrow -2 \le -2x < 1$

$\Rightarrow -1 < 2x \le 2$

$$\therefore \quad -\frac{1}{2} < x \le 1$$

If $-\frac{1}{2} < x < 0$

from (1) $\Rightarrow -3 = 5x - 2 \Rightarrow x = -\frac{1}{5}$

If $0 \le x < 1$

from (1) $\Rightarrow 0 = 5x - 2 \Rightarrow x = 2/5$ $\therefore$ 3 solution

If x = 1 (reject as $x \in I$ is already taken)

Q.26 **(C)**

0 < A < 1, 0 < B < 1, 0 < c < 1

$\Rightarrow$ 0 < A + B + C < 3

p = [A + B + C] = 2, q = [A] + [B] + [C] = 0

Maximum value of p – q means maximum value of p and minimum value of q = 2 – 0 = 2

JEE ADVANCED

MCQ/COMPREHENSION/ COLUMN MATCHING

Q.1 **(B,C)**

$f(x) = \ell n(\sin^{-1}(\log_2 x))$

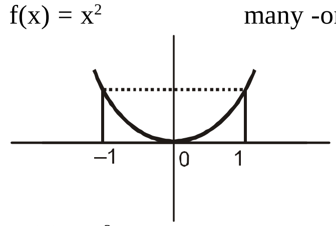
Domain $0 < \log_2 x \le 1$, $x \in (1, 2]$

Range $\left(-\infty,\ \ell n\frac{\pi}{2}\right]$

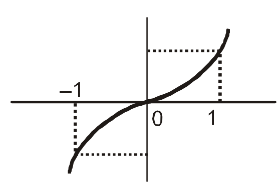
Q.2 (B,D)

Domain $D \in [-1, 1]$

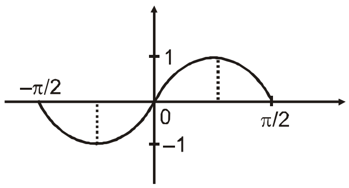
(A) $f(x) = x^2$ many -one



(B) $g(x) = x^3$ one - one



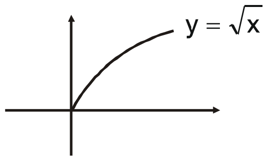
(C) $h(x) = \sin 2x$ many - one



(D) $k(x) = \sin\left(\frac{\pi x}{2}\right)$ one-one function

Q.3 (**A,D**)

$f : I^+ \to R,\ f(x) = \sqrt{x}$



Range = $\{y : y = \sqrt{n},\ n \in I^+\}$

$\therefore$ function is not onto but one-one

Q.4 (B,C,D)

$f : R \to [-1, 1]$

$$f(x) = \sin\left(\frac{\pi}{2}[x]\right) = \begin{cases} -1 & , \quad -1 \le x < 0 \\ 0 & , \quad 0 \le x < 1 \\ 1 & , \quad 1 \le x \le 2 \end{cases}$$

Many - one function

into function

Also $f(x + 4) = \sin\left(\frac{\pi}{2}[x+4]\right)$

$= \sin\left(2\pi + \frac{\pi}{2}[x]\right)$

$= \sin\left(\frac{\pi}{2}[x]\right)$

$= f(x)$ and hence periodic

Q.5 (**A,B,C**)

(A) $f(x) = \cos(\cos^{-1} x) = x,\ x \in [-1, 1]$

odd function

(B) $f(x + \pi) = \cos(\sin(x + \pi)) + \cos(\cos(x + \pi))$

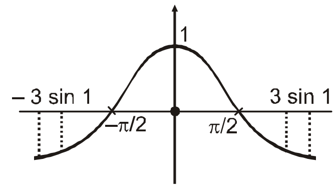
$f(x + \pi) = \cos(\sin x) + \cos(\cos x) = f(x)$

$$f\left(x + \frac{\pi}{2}\right) = \cos\left(\sin\left(x + \frac{\pi}{2}\right)\right) + \cos\left(\cos\left(x + \frac{\pi}{2}\right)\right)$$

$= \cos(\cos x) + \cos(\sin x)$

$= f(x)$

fundamental period = $\frac{\pi}{2}$



(C) $f(x) = \cos(3 \sin x),\ x \in [-1, 1]$

$-3 \sin 1 \le 3 \sin x \le 3 \sin 1$

$\Rightarrow \cos(3 \sin 1) \le \cos(3 \sin x) \le 1$

$\therefore$ Range is $[\cos(3 \sin 1), 1]$

Q.6 (**B**)

$f : R \to R,\quad f(x) = x^3 + ax^2 + bx + c$

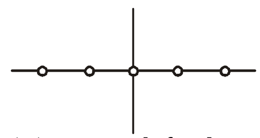
$f'(x) = 3x^2 + 2ax + b$

$D \le 0$ or $4a^2 - 12b \le 0$

or $a^2 \le 3b$

Q.7 (**A,B,C,D**)

$$f(x) = \frac{\sin(\pi[x])}{\{x\}} = 0,\ x \notin I$$



(A) By graph fundamental period is one

(B) $f(-x) = 0 = f(x)$

$\therefore$ even function

(C) Range $y \in \{0\}$

(D) $y = \text{sgn}\left(\text{sgn}\frac{\{x\}}{\sqrt{\{x\}}}\right) - 1,\ x \notin I$

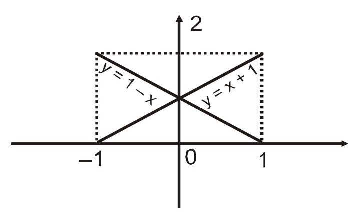
$y = \text{sgn}(1) - 1 \Rightarrow y = 1 - 1$

$y = 0,\ x \notin I$ Identical to $f(x)$

Q.8 (**A,B**)

$f ; [-1, 1] \to [0, 2]$

for onto function



Range = codomain

only two linear functions possible as show in graph.

Q.9 (**A,B,C**)

By definition

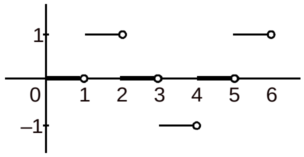
Q.10 (**B,C,D**)

$f : R \to [-1, 1]$

$0 \le x < 1;\quad y = 0$

$1 \le x < 2;\quad y = 1$

$2 \le x < 3$; $y = 0$



$3 \le x < 4$; $y = -1$
$4 \le x < 5$; $y = 0$
$5 \le x < 6$; $y = 1$

Q.11 **(A,C)**

$f(x) = \dfrac{1-x}{1+x}$, $0 \le x \le 1$

$g(x) = 4x(1-x), 0 \le x \le 1$

$$fog(x) = \frac{1-g(x)}{1+g(x)} = \frac{1-4x(1-x)}{1+4x(1-x)} = \frac{1-4x+4x^2}{1+4x-4x^2}$$

$gof(x) = 4f(x) \cdot (1 - f(x))$

$$= 4\left(\frac{1-x}{1+x}\right)\left(1-\left(\frac{1-x}{1+x}\right)\right)$$

$$= \frac{8x(1-x)}{(1+x)^2}$$

Q.12 **(A,B,C)**

$f(x) = \sin^4 3x + \cos^4 3x$

period = L.C.M. $\left(\dfrac{\pi}{3}, \dfrac{\pi}{3}\right) = \dfrac{\pi}{3}$

For fundamental period

$$f\left(x+\frac{\pi}{6}\right) = \sin^4\left(3\left(x+\frac{\pi}{6}\right)\right) + \cos^4\left(3\left(x+\frac{\pi}{6}\right)\right)$$

$= \cos^4 3x + \sin^4 3x = f(x)$

fundamental period = $\dfrac{\pi}{6}$

Q.13 (A,B,D)

(A) $f(x) = [x + 1] = [x] + 1$
non periodic
(B) $f(x) = \sin x^2$ non periodic
(C) $f(x) = \sin^2 x$ periodic with period π
(D) $f(x) = \sin^{-1} x$ monotonic $\Rightarrow$ non-periodic

Q.14 **(A,D)**

$f(x) = \sin x + \tan x + \text{sgn}(x^2 - 6x + 10)$
$f(x) = \sin x + \tan x + \text{sgn}((x - 3)^2 + 1)$
$f(x) = \sin x + \tan x + 1$
period = L.C.M. $(2\pi, \pi) = 2\pi$
fundamental period = 2π

Q.15 (B,C,D)

(A) $f(x) = e^{\ell n(\sec^{-1} x)} = \sec^{-1}x$;
$\sec^{-1}x > 0$ but not equal to zero, so $x \ne 1$
$x \in (-\infty, -1] \cup (1, \infty)$
$g(x) = \sec^{-1}x$, $x \in (-\infty, -1] \cup [1, \infty)$
non-identical functions

(B) $f(x) = \tan(\tan^{-1} x) = x$, $x \in R$
$g(x) = \cot(\cot^{-1} x) = x$, $x \in R$
identical functions

(C) $f(x) = \text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

$g(x) = \text{sgn}(\text{sgn } x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

Identical functions

(D) $f(x) = \cot^2 x \cdot \cos^2 x$,
$x \in R - \{n\pi\}$, $n \in I$
$g(x) = \cot^2 x - \cos^2 x$
$= \cot^2 x(1 - \sin^2 x)$
$= \cot^2 x \cdot \cos^2 x$
$x \in R - \{n\pi\}$, $n \in I$

Identical functions

Comprehension # 1 (Q. No. 16 to 18)

Q. 16 (A)

$f'(x) = x^2 + x + a$
$f'(x) \ge 0$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(a-\frac{1}{4}\right) \ge 0$$

Hence $a \ge \dfrac{1}{4}$

Q.17 (B)

If $a = -1$
$f'(x) = x^2 + x - 1$
$D = 1 + 4 > 0$
$\Rightarrow$ many one function and cubic polynomial have range $(-\infty, \infty)$ hence onto function

Q.18 (A)

For f(x) to be invertible f(x) should be one-one that is $f'(x) \ge 0$,

$$\Rightarrow a \in \left[\frac{1}{4}, \infty\right)$$

Comprehension # 2 (Q. No. 19 to 21)

Q.19 (A)

$f_2(x) = \frac{1}{2} \Rightarrow \left|f(|x|)\right| = \frac{1}{2}$

$\Rightarrow f(|x|) = \pm \frac{1}{2}$

$\Rightarrow |x| = \sqrt{e}, \sqrt{\frac{1}{2}}$

Q.20 (D)

$f(|x|) = \left|f(|x|)\right|$

$\Rightarrow f(|x|) \geq 0$

Q.21 (C)

Range of $f_3(x) = [-1, 1]$

$\Rightarrow \log_{27}(f_3(x)+2) = [\log_{27}1, \log_{27}3]$

Comprehension # 3 (Q. No. 22 to 24)

Q.22 (C)

Q.23 (A)

Q.24 (A)

Sol. (22 to 24)

Period of $e^{\tan\left\{\frac{x}{4}\right\}}$ is 4

$\cos\pi\left(\frac{(1-2[x])}{2}\right) = 0 \quad \forall\ x \in R$

Period of $\sin\left(\frac{\pi[x]}{2}\right)$ is 4

$\therefore$ Period of f(x) is 4

$\therefore$ p = 4 then $y = \sqrt{8 + 2[x] - [x]^2}$

$\therefore\ -[x]^2 + 2[x] + 8 \geq 0$

$\therefore\ [x]^2 - 2[x] - 8 \leq 0$

i.e., $([x] - 4)([x] + 2) \leq 0$

$\therefore\ -2 \leq [x] \leq 4$

$\therefore\ -2 \leq x < 5$

$\therefore$ q = – 2 ,

r = 5

$\therefore$ r – q – 1 = 5 + 2 – 1 = 6

$f_2(x) = \begin{cases} x+2, & x \geq 0 \\ 2-x, & x < 0 \end{cases}$

$f_2(f_2(x)) = \begin{cases} 2+f_2(x), & f_2(x) \geq 0 \\ 2-f_2(x), & f_2(x) < 0 \end{cases}$

=

$\begin{cases} 2+x+2, & x+2 \geq 0, & x \geq 0 \\ 2-(x+2), & x+2 < 0, & x \geq 0 \\ 2+2-x, & 2-x \geq 0, & x < 0 \\ 2-(2-x), & 2-x < 0, & x < 0 \end{cases}$

$= \begin{cases} 4+x, & x \geq 0 \\ 4-x, & x < 0 \end{cases}$

Range of $f_2(f_2(x))$ is $[4, \infty) \cup (4, \infty) = [4, \infty) = [p, \infty)$

Q.25 (A)–R; (B)–S; (C)–P; (D)–Q

(A) $-1 \leq \frac{x+1}{x} \leq 1 \Rightarrow \frac{1}{x} \leq 0$ & $\frac{2x+1}{x} \geq 0$

(B) $\frac{x^2+3x-2}{x+1} \geq 1 \Rightarrow \frac{x^2+2x-3}{x+1} \geq 0$

(C) $\frac{x-1}{2} > 0$ & $\frac{x-1}{2} \neq 1$

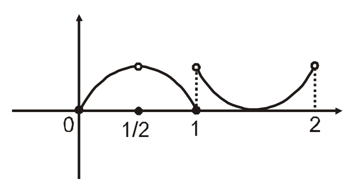
(D) $\sqrt{x^2+12}$ $2x > 0$

Q.26 (A) $\to$ (q, r), (B) $\to$ (q, r), (C) $\to$ (q), (D) $\to$ (s)

(A) $y = \sin\left(2\pi t + \frac{\pi}{3}\right) + 2\sin\left(3\pi + \frac{\pi}{4}\right) + 3\sin(5\pi t)$

LCM $\left(\frac{2\pi}{2\pi}, \frac{3\pi}{2\pi}, \frac{5\pi}{2\pi}\right)$ = LCM = $\left(1, \frac{2}{3}, \frac{2}{5}\right)$ = 2

(B) $y = \{\sin \pi x\}$ period = 2



For $x \in \left(0, \frac{1}{2}\right)$ function is one-one

For $x \in (0, 2)$ and $x \in (0, 8)$ function is money - one

(C) $y = \frac{1}{2}\left(\frac{\left|\sin\frac{\pi}{4}x\right|}{\cos\frac{\pi}{4}x} + \frac{\sin\frac{\pi}{4}x}{\left|\cos\frac{\pi}{4}x\right|}\right)$

(D) Since f(x) is bijective,

$\therefore$ f(0) = 0 or 2 but f(0) = 0 $\Rightarrow$ c = 0

(which is not true) $\therefore$ f(0) = 2 & f(2) = 0

Q.27 **(A)–S; (B)–R; (C)–P; (D)–Q**

(A) Graph below x-axis will shift to above the x-axis.

(B) f(|x|) is an even function. So graph on LHS of y-axis will below as same as that of RHS of y-axis

(C) f(|x|) is an even function. So graph on RHS of y-axis will below as same as that of LHS of y-axis

(D) $y = \begin{cases} 0 & \text{if } f(x) \ge 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$

Q.28 (A) → (q), (B) → (s), (C) → (p), (D) → (r),

(A) $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2} \Rightarrow x \in [0, 1]$

(B) $\sin^{-1}\sqrt{x} + \cos^{-1}\left(1-\sqrt{x^2}\right) = 0$

$\Rightarrow \cos^{-1}\sqrt{1-x^2} = -\sin^{-1}(x)$

$\Rightarrow x \in [-1, 0]$

(C) $g\left(\frac{1-x^2}{1+x^2}\right) = 2h(x)$

$\Rightarrow \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1} x$

$\Rightarrow x \in [0, \infty)$

(D) $h(x) + h(1) = h\left(\frac{1+x}{1-x}\right)$

$\Rightarrow \tan^{-1} x + \tan^{-1} = \tan^{-1}\left(\frac{1+x}{1-x}\right)$

$\Rightarrow x \in (-\infty, 1)$

NUMERICAL VALUE BASED

Q.1 [17]

$7x - x^2 - 6 \ge 0 \Rightarrow x^2 - 7(x) + 6 \le 0$

$(x-1)(x-6) \le 0 \Rightarrow x \in [1, 6]$ and

$\sin x + \cos x \ge 0 \Rightarrow \left[1, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 6\right]$

Q.2 [0]

$f(x) = \frac{(x^2+x+2)(x+1)}{(x^2+x+1)(x+1)}$; $x \in R - \{0\}$

$f(x) = \frac{x^2+x+2}{x^2+x+1}$; $x \in R - \{0, -1\}$

$y = \frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow (y-1)x^2 + (y-1)x + y - 2 = 0$

$y \ne 1$, $D \ge 0$

$(y-1)^2 - 4(y-1)(y-2) = 0$

$\Rightarrow 1 < y \le 7/3$

at $x = 0$ we get $y = 2$

& $y = 2 \Rightarrow 2 = \frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow x(x+1) = 0$

$\Rightarrow x = 0, -1$ but $x \ne 0, -1$

so $y \ne 2$

Range $\left(1, \frac{7}{3}\right] - \{2\}$.

Q.3 [2]

f & g are 2 distinct functions $[-1, 1] \to [0, 2]$ onto functions

So f & g are either $-x + 1$ or $x + 1$

Case-I $f(x) = -x + 1$; $g(x) = x + 1$

$h(x) = \frac{f(x)}{g(x)} = \frac{1-x}{1+x}$;

$h(h(x)) = \frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}} = x$

$h(1/x) = \frac{x-1}{x+1}$;

$h(h(1/x)) = \frac{1-\frac{x-1}{x+1}}{1+\frac{x-1}{x+1}} = \frac{1}{x}$

$\left|h(h(x)) + h\left(h\left(\frac{1}{x}\right)\right)\right| = |x + 1/x| > 2$ as domain does not contain point $x = \pm 1$

Case-II

$f(x) = 1 + x$; $g(x) = 1 - x$

$h(x) = \frac{1+x}{1-x}$;

$h(h(x)) = \frac{1+\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}} = -\frac{1}{x}$

$h(h(1/x)) = -x$; $\left|h(h(x)) + h\left(h\left(\frac{1}{x}\right)\right)\right| = |-x - 1/x| = (x + 1/x) > 2$

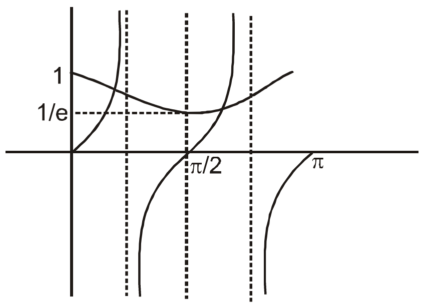
Q.4 [34]

$f(-x) = -[ax^7 + bx^3 + cx] - 5$; $f(-x) = -[f(x) + 5] - 5$

$f(-x) = -f(x) = -10$ put $x = 7$

$f(7) = -17$ so $f(7) + 17 \cos x = -17(\cos x - 1) \in [-34, 0]$

Q.5 **[20]**



Period of $e^{-\sin^2 x}$ is π

and that of tan2x is $\pi/2$

so number of solutions in $(0, \pi)$ is 2

Number of solutions in $[0, \pi]$ is 2

so number of solution in $[0, 10\pi] = 20$

Q.6 [22]

21 x

22 y

23 z

	case-I	**case-II**	**case-III**
$f(21) = x$	T	F	F
$f(22) \neq x$	F	T	F
$f(23) \neq y$	F	F	T

case-I $f(22) = x$, $f(23) = y$
then $f(21) = x$ is not true

case-II $f(23) = y$, $f(22) = z$, $f(21) = x$
not possible

case-III $f(22) = x$, $f(23) = z$, $f(21) = y$

$\therefore$ $f^{-1}(x) = 22$

Q.7 [3] or $\left[x = 1, \frac{-1 \pm \sqrt{5}}{2}\right]$

$$\frac{x^3 + 1}{2} = 2\sqrt[3]{2x - 1}$$

Let $f(x) = \frac{x^3 + 1}{2}$

$\Rightarrow f^{-1}(x) = \sqrt[3]{2x - 1}$

Equation becomes $f(x) = f^{-1}(x)$

$\Rightarrow f(x) = x$

$\Rightarrow \frac{x^3 + 1}{2} = x$

$\Rightarrow x^3 - 2x + 1 = 0$

$\Rightarrow (x - 1)(x^2 + x - 1) = 0$

$\Rightarrow x = 1, \frac{-1 \pm \sqrt{5}}{2}$

Allter :

Let $y = \sqrt[3]{2x - 1}$

$\Rightarrow y^3 - 2x + 1 = 0$ and $x^3 - 2y + 1 = 0$

$\Rightarrow (y^3 - 2x + 1) - (x^3 - 2y + 1) = 0$

$\Rightarrow (y - x)(y^2 + xy + x^2 + 2) = 0$

$\Rightarrow y = x$ or $y^2 + xy + x^2 + 2 = 0$

$\Rightarrow y = x$ or $(x + y)^2 + x^2 + y^2 + 4 = 0$

Putting $y = x$ in $y = \sqrt[3]{2x - 1}$, we get

$$x^3 - 2x + 1 = 0$$

Which yields the values $x = 1, \frac{-1 \pm \sqrt{5}}{2}$

KVPY

PREVIOUS YEAR'S

Q.1 (C)

$f(x) = ax^2 + bx + c$

given $f(1) = 0$

$\Rightarrow a + b + c = 0$

and $40 < f(6) < 50$

$\Rightarrow 40 < 36a + 6b + c < 50$

$\Rightarrow 40 < 35a + 5b < 50$

$\Rightarrow 8 < 7a + b < 10$

$7a + b = \text{integer} = 9$ .....(1)

and $60 < f(7) < 70$

$\Rightarrow 60 < 49a + 7b + c < 70$

$\Rightarrow 60 < 48a + 6b < 70$

$\Rightarrow 10 < 8a + b < 11.6$

$8a + b = \text{integer} = 11$ ......(2)

Solving (1) & (2)

$a = 2, b = -5, c = 3$

$\therefore f(x) = 2x^2 - 5x + 3$

$f(50) = 4753$

1000 t < f (50) < 1000(t + 1)
(1000 × 4) < 4753 < 1000 (4 + 1)
$\therefore$ t = 4

Q.2 (C)
Clearly f (π + x) + f (π – x) (every term contain cosine)

$$f\left(\frac{\pi}{5}\right)=f\left(\frac{9\pi}{5}\right),f\left(\frac{2\pi}{5}\right)=f\left(\frac{8\pi}{5}\right),f\left(\frac{3\pi}{5}\right)=f\left(\frac{7\pi}{5}\right)$$

$$f\left(\frac{4\pi}{5}\right)=f\left(\frac{6\pi}{5}\right)$$

$$T=f(0)-2\left[f\left(\frac{\pi}{5}\right)+f\left(\frac{3\pi}{5}\right)\right]$$

$$+2\left[f\left(\frac{2\pi}{5}\right)+f\left(\frac{4\pi}{5}\right)\right]-f(\pi)$$

f (0) – f (π) = 2 (1 + B + D)

$$f\left(\frac{\pi}{5}\right)+f\left(\frac{3\pi}{5}\right)=f\left(\frac{\pi}{5}\right)-f\left(\frac{4\pi}{5}\right)$$

$$=2\left(1+B\cos\frac{3\pi}{5}+D\cos\frac{\pi}{5}\right)$$

$$f\left(\frac{2\pi}{5}\right)+f\left(\frac{4\pi}{5}\right)=f\left(\frac{2\pi}{5}\right)-f\left(\frac{3\pi}{5}\right)$$

$$=2\left(1+B\cos\frac{6\pi}{5}+D\cos\frac{2\pi}{5}\right)$$

T $\Rightarrow$ contains only B, D terms

Q.3 (C)
$x \notin I$ & $[x] > 1$
$\Rightarrow x \in$ (2, 3) only option satisfy.

Q.4 (C)

$$f(x, A\cup B)=\begin{cases}1, \text{ if } & x\in A\cup B\\ 1, \text{ if } & x\notin A\cup B\end{cases}$$

$$\left.\begin{array}{l}\text{if } \; x\in A, \; x\in B\\ \text{if } \; x\in A, \; x\notin B\\ \text{if } \; x\notin A, \; x\in B\end{array}\right\}\Rightarrow f(x, A\cup B)=1\Rightarrow \text{None of the}$$

option (A, B, D) satisfy

$$\text{if } x\notin A, x\notin B\Rightarrow f(x, A\cup B)=0\Rightarrow C \text{ (only C satisfy)}$$

Q.5 (C)
$(2011)^n + {}^nC_1(2011)^{n-1} + {}^nC_2(2011)^{n-2} + ... + {}^nC_{n-1}(2011) + {}^nC_{n-1}$
$= (2011+1)^n - 1$

Q.6 (D)
f (x) $= x^9 (x^3 - 1) + x (x^3 - 1) + 1$ positive for $x \geq 1$ or $x \leq 0$
$= 1 - x + x^4 - x^9 + x^{12}$ positive for $x \in (0, 1)$
f (x) is always positive

Q.7 (2)

$$f(x)=\sqrt{4-\sqrt{2x+5}}$$

$4-\sqrt{2x+5}\geq 0$ $\qquad$ $2x + 5 \geq 0$
$\sqrt{2x+5}\leq 4$ $\qquad$ $x \geq -5/2$

$$x\leq\frac{11}{2}$$

$$x\in\left[-\frac{5}{2},\frac{11}{2}\right]$$

$$\text{mid point}=\frac{-5/2+11/2}{2}=\frac{3}{2}$$

Q.8 (B)
Only when $a_1 = a_2 = a_3$
In other cases f (x) will take both positive and negative values

Q.9 (B)

$$f(x)=\frac{x+1}{x-1}$$

$$f^2(x) = f(f(x)) = f\left(\frac{x+1}{x-1}\right)=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}=x$$

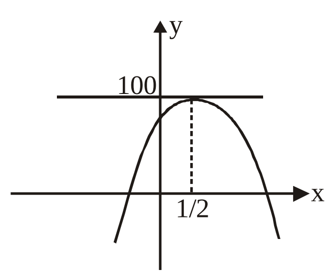
$$f^3(x) = f(x) = \frac{x+1}{x-1}$$

$f^4(x) = x$
$P = f(2).\ f^3(3)\ f^3(4)\ f^4(5)$

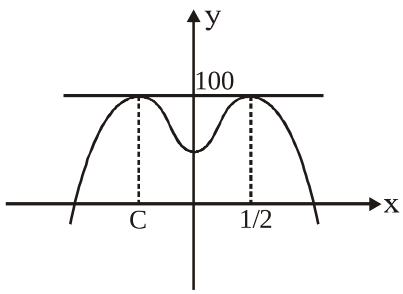
$$P = 3\times 3\times\frac{5}{3}\times 5 = 75$$

Multiple of P is 375

Q.10 (C)
Coefficient of highest degree term must be negative because if it is positive, then $x \to \infty$, $y \to \infty$ and it is not possible, since $f(x) \leq 100$.
Now, graph will be like



at least two real roots will be there, & if $x \neq \frac{1}{2}$, then f(x) < 100, it is not always true, as the graph can be like this also



Now, let the highest coefficients, it can have is 49

then, $f\left(\frac{1}{2}\right) = 49 + \frac{49}{2} + \frac{49}{2^2} + ...$

But the sum cannot be equal to 100.

Q.11 (B)
Domain of f(g(x)) is R
$\because$ $2 - \cos x - \cos^2 x \geq 0$
$(\cos x + 2)(\cos x - 1) \leq 0 \Rightarrow -2 \leq \cos x \leq 1$
$x \in R$
Domain of g(f(x)) is [–2, 1]

$\because \cos\left(\sqrt{2 - x - x^2}\right)$

$2 - x - x^2 \geq 0$
Domain of $f(g(x)^2)$ is R
$\because$ $2 - \cos^2 x - \cos^4 x \geq 0$
$(\cos^2 x + 2)(\cos^2 x - 1) \leq 0$
$-1 \leq \cos x \leq 1$
$x \in R$
Domian of $g(f^3(x))$ is Domain of g(f(x))
i.e., [–2, 1]

Q.12 (C)

$$f(x) + \left(x + \frac{1}{2}\right) f(1 - x) = 1$$

$$f(1 - x) + \left(1 - x + \frac{1}{2}\right) f(1 - (1 - x)) = 1$$

$$f(1 - x) + \left(\frac{3}{2} - x\right) f(x) = 1$$

$$\frac{1 - f(x)}{x - \frac{1}{2}} + \left(\frac{3}{2} - x\right) f(x) = 1$$

$$1 - f(x)\left(\frac{3}{2}x - x^2 + \frac{3}{2} - \frac{x}{2}\right) f(x) = x + \frac{1}{2}$$

$$f(x) + \left(x - x^2 - \frac{1}{4}\right) = x - \frac{1}{2}$$

$$f(x)\left(4x - 4x^2 - 1\right) = 4x - 2$$

$$f(x) = \frac{-2 + 4x}{4x - 4x^2 - 1}$$

$$2f(0) + 3f(1) = 2\left(\frac{-2 + 0}{0 - 0 - 1}\right) + 3\left(\frac{-2 + 4}{4 - 4 - 1}\right)$$

$$= +4 + \frac{3(+2)}{-1} = +4 - 6 = -2$$

Alternate
Put x = 0

$$f(0) + \frac{1}{2} f(1) = 1 \Rightarrow 2f(0) + f(1) = 2$$

put x = 1

$$f(1) + \frac{3}{2} f(0) = 1 \Rightarrow 2f(1) + 3f(0) = 2$$

solving above f(0) = 2 and f (1) = – 2
$\therefore$ 2f(0) + 3f(1) = 4 – 6 = – 2

Q.13 (B)

$$S = -\left[\frac{\lambda - a_n}{a_n} + \frac{\lambda - a_{n-1}}{a_{n-1}} + ... + \frac{\lambda - a_1}{a_1}\right]$$

$$\forall \lambda = a_1 + a_2 + ... + a_n$$

$$S = -\left[\left(a_1 + a_2 + ... + a_n\right)\left(\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right) - n\right]$$

$$S = n - \left(a_1 + a_2 + ... + a_n\right)\left(\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right)$$

from A.H. $\geq$ H.M.

$$\left(a_1 + a_2 + ... + a_n\right)\left(\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right) \geq n^2$$

$$\boxed{S \leq -n(n-1)}$$

Q.14 (D)
Q.15 (C)
Case (1) n = even

$$\frac{A_n}{A_{n-1}} = \frac{{}^nC_{n/2}}{{}^{n-1}C_{\frac{n-1-1}{2}}} = 2$$

so for all n even given relation is true.
Case (2) n = odd

$$\frac{A_n}{A_{n-1}} = \frac{{}^nC_{\frac{n-1}{2}}}{{}^{n-1}C_{\frac{n-1-1}{2}}} = \frac{2n}{n+1}$$

which satisfies only for n = 19

Q.16 (D)

(A) $[x + y] \leq [x] + [y]$

let x = 0.1

y = 0.9

$[0.1 + 0.9] \leq [0.1] + [0.9]$

$1 \leq 0 + 0$ wrong

(B) $[xy] \leq [x][y]$

$x = 2; y = \frac{1}{2}$

$\left[2.\frac{1}{2}\right] \leq [2]\left[\frac{1}{2}\right]$

$\Rightarrow 1 \leq 0$ wrong

(C) $\left[2^x\right] \leq 2^{[x]}$

$x = 099\left[2^{0.99}\right] \leq 2^{[0.99]}$

$\left[2^{0.99}\right] \leq 2^o = 1$ wrong

(D) $\left[\frac{x}{y}\right] \leq \frac{[x]}{[y]}$

given x, y ≥ 1

if $x < y\left[\frac{x}{y}\right] = 0 \quad 0 \leq \frac{[x]}{[y]}$ true

if $x \geq y\left[\frac{x}{y}\right] \leq \frac{[x]}{[y]}$ always true

Q.17 (D)

f (x y) = f (x) + f (y)

$\Rightarrow$ f (x) = $\log_a x$

So, f (12) = 24

$\Rightarrow \log_a 12 = 24$

$\Rightarrow 12 = a^{24}$ & f (8) = 15

$\Rightarrow \log_a 8 = 15$

$\Rightarrow 8 = a^{15} \Rightarrow 2 = a^5$

So, f (48) = $\log_a 48 = \log_a 12 + \log_a 4$

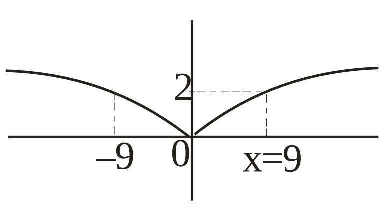
$= \log_a 12 + \log_a 2^2$

= 24 + 2 ·5

= 34

Q.18 (B)

graph of given function actually look like this



Clear from graph option(B) is right

Q.19 (D)

$$f(x) = \frac{\{x\}}{1+[x]^2}$$

$$f(x) = \begin{cases} \frac{x+1}{2}; & -1 \leq x < 0 \\ x; & 0 \leq x < 1 \\ \frac{x-1}{2}; & 1 \leq x < 2 \\ \frac{x-2}{5} & 2 \leq x < 3 \\ & \vdots \\ & \text{So – on} \end{cases}$$

Now check accordingly.

Q.20 (B)

Let h (x) = 9 (f (x))

and n (x) is onto (given)

$\therefore$ Co-domain of h (x) = Range of h (x)

Range of h (x) = [0, 2]

Range of h (x) = 9 (f (x)) it means g is giving [0, 2] which is also co- domain of g.

So, g must be onto.

Now, Domain of g = [– 1, 1] which must be range of f.

But, co-domain of f = [– 1, 1]

So, f must be onto

Q.21 (D)

Only condition that g(x) should satisfy for gof (x) = $x \forall x \in [0,1]$ is that g(x) should attain all values in [0, 1] when range of f (x) a subset of (– 1, 1) is used as image for g (x). Thus there can be infinite such function g (x) with domain [– 1, 1] and range [0, 1]

Q.22 (D)

$f(x^2) = f(x^3)$

$\Rightarrow f(\alpha) = f(\alpha^{2/3}) = f(\alpha^{4/9}) = ... = (\alpha^{2^n/3^n}) = f(\alpha^0)$ as $n \to \infty$

$\Rightarrow$ f(a) = f(1) $\Rightarrow$ f(x) is constant function

$\Rightarrow$ f(x) is differentiable and even function

Q.23 (B)

$f(x) = x^6 - 2x^5 + x^3 + x^2 - x - 1 = (x^2 - x)(x^4 - x^3 - x^2 - 1) + (2x^2 - 2x - 1)$

$\Rightarrow f(a) = 2a^2 - 2a - 1$

$\Rightarrow f(a) + f(b) + f(c) + f(d) = 2(a^2 + b^2 + c^2 + d^2) - 2(a + b + c + d) - 4$

= 2[1 – 2(–1)] – 2(1) – 4 = 0

Q.24 (B)

$(x - [x])(x^2 + [x]^2 + x[x])$

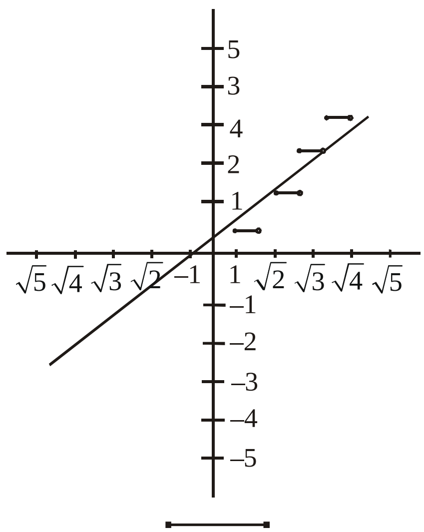
$= (x - [x])(x^2 + [x]^2 - 2x[x])$

$\Rightarrow (x - [x])(3x[x]) = 0$

$\Rightarrow x = 0$ or $[x] = 0$ or $x = [x]$

$\Rightarrow x \in Z \cup [0, 1)$

Q.25 (C)



From the graph it is clear that the equation has no-solution.

Q.26 (B)

$$\frac{2^{2020}+1}{2^{2018}+1} = \frac{4.2^{2018}+1}{2^{2018}+1} = \frac{4\left(2^{2018}+1\right)-3}{2^{2018}+1}$$

$$4 - \frac{3}{2^{2018}+1} - t, 3 < t < 4$$

Now $\left[\frac{2^{2020}+1}{2^{2018}+1}\right] = 3$

similarly $\frac{3^{2020}+1}{2^{2018}+1} = \frac{9.3^{2018}+1}{3^{2018}+1} = 9 - \frac{8}{3^{2018}+1}$

$$\left[\frac{3^{2020+1}}{3^{2018}+1} = 8\right]$$

similarly $\left[\frac{n^{2020}+1}{2^{2018}+1}\right] = n^2 - 1$

$(2^2 - 1) + (3^2 - 1) + (4^2 - 1) + (5^2 - 1) + (6^2 - 1)$

$= 3 + 8 + 15 + 24 + 35 = 85$

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (3)

$f(n + 1) = f(n) + 1$

$f(2) = 2f(1)$

$f(3) = 3f(1)$

$f(4) = 4f(1)$

....

$f(n) = nf(1)$

$f(x)$ is one-one

Q.2 (4)

$$f(x) = \frac{5^x}{5 + 5^x}$$

$$f(2 - x) = \frac{5^{2-x}}{5 + 5^{2-x}}$$

$$= \frac{25}{5.5^x + 25} = \frac{5}{5^x + 5}$$

$f(x) + f(2 - x) = 1$

$$\text{Now}\left(f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right)\right) + \left(f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right)\right) + \ldots + $$

$$\left(f\left(\frac{19}{12}\right) + f\left(\frac{21}{20}\right)\right) + f\left(\frac{20}{20}\right)$$

$$= 1 \times 19 + \frac{1}{2} = \frac{39}{2}$$

Q.3 (2)

$$af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \quad .....(i)$$

$$x \rightarrow \frac{1}{x}$$

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{\beta}{x} + \beta x \quad ......(ii)$$

(i) + (ii)

$$(a + \alpha)\left[f(x) + f\left(\frac{1}{x}\right)\right] = \left(x + \frac{1}{x}\right)(b + \beta)$$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

Q.4 (1)

$$g(2)=\lim_{x\to 2}\frac{(x-2)(x+1)}{(2x+3)(x-2)}=\frac{3}{7}$$

For domain of fog (g)

$$\left|\frac{x^2-x-2}{2x^2-x-6}\right|\le 1 \Rightarrow (x+1)^2\le(2x+3)^2$$

$\Rightarrow$ (3x+4) (x+2)≥0

$x \in (-\infty, -2)\cup\left(-\frac{4}{3},\infty\right]$

Q.5 (2)

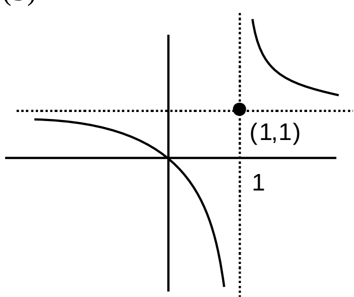
g(f(x))=f(x)

$\Rightarrow$ g(x)=x, when x is even

$\therefore$ So total number of functions from A to A

$= 10^5\times1=10^5$

Q.6 (3)



f(g(x)) = 2g(x) – 1

$$= 2\frac{\left(x-\frac{1}{2}\right)}{x-1} = \frac{x}{x-1}$$

$$f(g(x)) = 1+\frac{1}{x-1}$$

one-one, into

Q.7 (3)

Given = y = $5^{(\log_a x)}$ $f(x)$

Interchanging x & y for inverse

= y = $5^{(\log_a y)} = y^{(\log_a 5)}$

option (1) or option (2)

Further, from given relation

$\log_5 y = \log_a x$

$\Rightarrow x = a^{(\log_5 y)} = y^{(\log_5 a)}$

$\Rightarrow x = y^{\left(\frac{1}{\log_a 5}\right)} = f^{-1}(y)$

option (3)

Q.8 (2)

$$(f)x=\frac{\operatorname{cosec}^{-1}x}{\sqrt{\{x\}}}$$

Domain $\in(-\infty,-1]\cup[1,\infty)$

$\{x\}\ne 0$ so $x\ne$ integers

Q.9 (3)

$f(x)+g(x)=\sqrt{x}+\sqrt{1-x}$, domain [0, 1]

$f(x)-g(x)=\sqrt{x}-\sqrt{1-x}$, domain [0, 1]

$g(x)-f(x)=\sqrt{1-x}-\sqrt{x}$, domain [0, 1]

$\frac{f(x)}{g(x)}=\frac{\sqrt{x}}{\sqrt{1-x}}$, domain [0, 1)

$\frac{g(x)}{f(x)}=\frac{\sqrt{1-x}}{\sqrt{x}}$, domain (0, 1]

So, common domain is (0, 1)

Q.10 (3)

$f(x) = y = \frac{x-2}{x-3}$

$\therefore x = \frac{3y-2}{y-1}$

$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$

& g(x) = y = 2x – 3

$\therefore x = \frac{y+3}{2}$

$\therefore g^{-1}(x) = \frac{x+3}{2}$

$\therefore f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\therefore x^2 - 5x + 6 = 0$ ⟨ x_1, x_2

$\therefore$ sum of roots

$x_1 + x_2 = 5$

Q.11 **(1)**

Q.12 [**720]**

Q.13 (1)

Q.14 (2)

Q.15 (3)

Q.16 [490]

Q.17 (3)

Q.18 (3)

Q.19 **(2)**

Q.20 (4)

Q.21 [26]

Q.22 (2)

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (A)

$f(x) = x^2$; $g(x) = \sin x$

$\Rightarrow gof(x) = \sin x^2$

$\Rightarrow gogof(x) = \sin(\sin x^2)$

$\Rightarrow (fogogof)(x) = (\sigma in (\sin x^2))^2 = \sin^2(\sin x^2)$

Now $\sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow \sin(\sin x^2) = 0, 1$

$\Rightarrow \sin x^2 = n\pi, (4n+1)\frac{\pi}{2}$; $\eta \in I$

$\Rightarrow \sin x^2 = 0$

$\Rightarrow x^2 = n\pi$

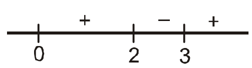
$\Rightarrow x = \pm\sqrt{n\pi}$; $n \in W$

Q.2 (B)

$F : [0, 3] \to [1, 29]$

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x-2)(x-3)$



in given domain function has local maxima, it is many-one

Now at $x = 0$ $f(0) = 1$

$x = 2$ $f(2) = 16 - 60 + 72 + 1 = 29$

$x = 3$ $f(3) = 54 - 135 + 108 + 1 = 163 - 135 = 28$

Has range = [1, 29]

Hence given funχτιon is onto

Q.3 (AB)

NOTE : Since a functional mapping can't have two images for pre-image 1/3, so this is ambiguity in this question perhaps the answer can be A or B or AB or marks to all.

$\cos 4\theta = \frac{1}{3} \Rightarrow 2\cos^2 2\theta - 1 = \frac{1}{3} \Rightarrow \cos^2 2\theta = \frac{2}{3}$

$\Rightarrow \cos 2\theta = \pm\sqrt{\frac{2}{3}}$

Now $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta} = \frac{1+\cos 2\theta}{\cos 2\theta} = 1 + \frac{1}{\cos 2\theta}$

$\Rightarrow f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$

Q.4 (A,B,C)

(i) $f(-x) = -f(x)$ so it is odd function

(ii) $f'(x) = 3(\log(\sec x + \tan x))^2 \frac{1}{(\sec x + \tan x)}(\sec x \tan x + \sec^2 x) > 0$

(iii) Range of f(x) is R as $f\left(-\frac{\pi}{2}\right) \Rightarrow -\infty$

Q.5 (B,C,D)

$\alpha = 3\sin^{-1}\frac{6}{11} > 3\sin^{-1}\frac{6}{12}$ and

$\beta = 3\cos^{-1}\frac{4}{9} > 3\cos^{-1}\frac{4}{8}$

$\Rightarrow \alpha > \frac{\pi}{2}$ &

$\beta > \pi$

$\Rightarrow \alpha + \beta > \frac{3\pi}{2}$

Q.6 [19.00]

$f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2}$

$= \frac{4^x}{4^x+2} + \frac{4/4^x}{\frac{4}{4^x}+2}$

$= \frac{4^x}{4^x+2} + \frac{4}{4+2.4^x}$

$= \frac{4^x}{4^x+2} + \frac{2}{2+4^x}$

$= 1$

so, $f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + ... + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right)$

$= 19 + f\left(\frac{1}{2}\right) - f\left(\frac{1}{2}\right) = 19$

Trigonometric Function

EXERCISES

ELEMENTRY

Q.1 (2)

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)=-\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}$$

Q.2 (1)

$$\cos^{-1}(\cos 12)-\sin^{-1}(\sin 14)\Rightarrow 12-14=-2\cdot$$

Q.3 (3)

Given that $\tan^{-1}x-\tan^{-1}y=\tan^{-1}A$

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}A$$

Hence $A=\dfrac{x-y}{1+xy}$.

Q.4 (2)

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\cos^{-1}\left\{\cos\left(\pi+\frac{\pi}{6}\right)\right\}$$

$$=\cos^{-1}\left(-\cos\frac{\pi}{6}\right)=\pi-\cos^{-1}\cos\frac{\pi}{6}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}.$$

Q.5 (2)

Obviously $x=\sin(\theta+\beta)$ and $y=\sin(\theta-\beta)$

$\therefore 1+xy=1+\sin(\theta+\beta)\sin(\theta-\beta)$

$=1+\sin^2\theta-\sin^2\beta=\sin^2\theta+\cos^2\beta$

Q.6 (1) $\tan^{-1}\left[\dfrac{\cos x}{1+\sin x}\right]=\tan^{-1}\left[\dfrac{\sin(\pi/2-x)}{1+\cos(\pi/2-x)}\right]$

$$=\tan^{-1}\left[\frac{2\sin(\pi/4-x/2)\cos(\pi/4-x/2)}{2\cos^2(\pi/4-x/2)}\right]$$

$$=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{\pi}{4}-\frac{x}{2}$$

Q.7 (3)

Let $\tan^{-1}2=\alpha\Rightarrow\tan\alpha=2$

and $\cot^{-1}3=\beta\Rightarrow\cot\beta=3$

$\sec^2(\tan^{-1}2)+\text{cosec}^2(\cot^{-1}3)$

$=\sec^2\alpha+\text{cosec}^2\alpha=1+\tan^2\alpha+1+\cot^2\alpha$

$=2+(2)^2+(3)^2=15$

Q.8 (1)

Given, $\tan^{-1}x=\sin^{-1}\left[\dfrac{3}{\sqrt{10}}\right]$

$\Rightarrow x=\tan\left\{\sin^{-1}\left[\dfrac{3}{\sqrt{10}}\right]\right\}=\tan\{\tan^{-1}3\}$

$\Rightarrow x=3.$

Q.9 (3)

$$\sin\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]=\cos\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$=\cos\cos^{-1}\sqrt{1-\frac{3}{4}}=\frac{1}{2}$$

Q.10 (2)

$$\cot^{-1}3+\text{cosec}^{-1}\sqrt{5}=\cot^{-1}3+\cot^{-1}2$$

$$=\cot^{-1}\left(\frac{3\times 2-1}{3+2}\right)=\cot^{-1}(1)=\frac{\pi}{4}.$$

$$=\cot^{-1}\left(\frac{3\times 2-1}{3+2}\right)=\cot^{-1}(1)=\frac{\pi}{4}.$$

Q.11 (1)

$$\cos^{-1}\left[\cos\frac{5\pi}{3}\right]+\sin^{-1}\left[\frac{\cos 5\pi}{3}\right]=\frac{\pi}{2}$$

$\left(\because \sin^{-1}x+\cos^{-1}x=\dfrac{\pi}{2}\right)$.

Q.12 (4)

$$\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]-\sin^{-1}\left[\frac{1}{2}\right]=60^o-30^o=30^o.$$

Q.13 (4)

$$\tan\left[\tan^{-1}\frac{1}{2}-\tan^{-1}\frac{1}{3}\right]=\tan\left[\tan^{-1}\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{6}}\right]$$

$$=\tan\tan^{-1}\left(\frac{1}{6}\times\frac{6}{7}\right)=\frac{1}{7}.$$

Q.14 (2)

$$\cos\left(\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}+\cos^{-1}\frac{1}{5}\right)=\cos\left(\frac{\pi}{2}+\cos^{-1}\frac{1}{5}\right)$$

$$=-\sin\left(\cos^{-1}\frac{1}{5}\right)=-\sin\left(\sin^{-1}\sqrt{\frac{24}{25}}\right)=-\frac{2\sqrt{6}}{5}$$

Q.15 (1)

$$\sin^{-1}x+\sin^{-1}\frac{1}{x}+\cos^{-1}x+\cos^{-1}\frac{1}{x}$$

$$=\{\sin^{-1}(x)+\cos^{-1}(x)\}+\left\{\sin^{-1}\left(\frac{1}{x}\right)+\cos^{-1}\left(\frac{1}{x}\right)\right\}$$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

Q.16 (1)

$\sin^{-1}\frac{4}{5} = \tan^{-1}\frac{4}{3}, 2\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{3}{4} = \cot^{-1}\frac{4}{3}$

and $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

Q.17 (1)

Let $\cos^{-1}x = \theta$. Then $x = \cos\theta$

$\tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{\frac{1}{x^2} - 1} = \sqrt{\frac{1-x^2}{x}}$

$\therefore \tan(\cos^{-1}x) = \tan\theta = \frac{\sqrt{1-x^2}}{x}$.

Q.18 (4)

$\tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right] = \tan\left[\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}} - \tan^{-1}(1)\right]$

$= \tan\left[\tan^{-1}\frac{5}{12} - \tan^{-1}(1)\right] = \tan\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right) = -\frac{7}{17}$

Q.19 (2)

Let $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

Now, $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \frac{1}{2}\cos^{-1}\cos 2\theta = \frac{2\theta}{2} = \theta = \tan^{-1}\sqrt{x}$

Q.20 (2)

$\sin\left(4\tan^{-1}\frac{1}{3}\right) = \sin\left[2\tan^{-1}\left(\frac{2/3}{1-(1/9)}\right)\right]$

$= \sin\left[2\tan^{-1}\frac{3}{4}\right] = \sin\sin^{-1}\left(\frac{2\times(3/4)}{1+(9/16)}\right)$

$= \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$

$\left(\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right)$

Q.21 (2)

$\sin\left[2\tan^{-1}\left(\frac{1}{3}\right)\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \sin\left[\tan^{-1}\frac{2/3}{1-1/9}\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \sin[\tan^{-1}3/4] + \cos[\tan^{-1}2\sqrt{2}]$

$= \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$.

Q.22 (4)

$\sin^{-1}2x = \sin^{-1}x - \sin^{-1}\frac{\sqrt{3}}{2}$

$\sin^{-1}2x = \sin^{-1}\left(x\sqrt{\left(1-\frac{3}{4}\right)} - \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$

$2x = \left(\frac{x}{2} - \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$

$\frac{\sqrt{3}}{2}\sqrt{1-x^2} = \frac{x}{2} - 2x = \frac{-3x}{2}$

$\frac{3(1-x^2)}{4} = \frac{9x^2}{4}$

$\Rightarrow 3 - 3x^2 = 9x^2 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$.

Q.23 (1)

$\sin\left[3\sin^{-1}\frac{1}{5}\right] = \sin\left[\sin^{-1}\left\{3\left(\frac{1}{5}\right) - 4\left(\frac{1}{5}\right)^3\right\}\right]$

$= \sin\left[\sin^{-1}\left\{\frac{3}{5} - \frac{4}{125}\right\}\right] = \sin\left[\sin^{-1}\left(\frac{75-4}{125}\right)\right]$

$= \sin\left[\sin^{-1}\frac{71}{125}\right] = \frac{71}{125}$

Q.24 (3)

$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$

$\Rightarrow \tan^{-1}(1+x) = \cot^{-1}(1-x)$

$\Rightarrow \tan^{-1}(1+x) = \tan^{-1}\left(\frac{1}{1-x}\right)$

$\Rightarrow 1+x=\dfrac{1}{1-x} \Rightarrow 1-x^2=1 \Rightarrow x=0$

Q.25 (3)

$x+y=\tan^{-1}33$

$\Rightarrow y=\tan^{-1}33-\tan^{-1}3$

$=\tan^{-1}\dfrac{33-3}{1+99}=\tan^{-1}\dfrac{30}{100} \Rightarrow y=\tan^{-1}(0.3)$

Q.26 (3)

The given equation may be written as

$\tan^{-1}x+\cot^{-1}x+\cot^{-1}x=\dfrac{2\pi}{3}$

$\Rightarrow \cot^{-1}x=\dfrac{2\pi}{3}-\dfrac{\pi}{2}=\dfrac{\pi}{6} \Rightarrow x=\sqrt{3}$.

Q.27 (2)

$\because \cot^{-1}\dfrac{1}{2}=\cos^{-1}\dfrac{1}{\sqrt{5}}$

Hence given equation can be written as

$\sin^{-1}x+\cos^{-1}\dfrac{1}{\sqrt{5}}=\dfrac{\pi}{2} \quad \Rightarrow x=\dfrac{1}{\sqrt{5}}$

Q.28 (2)

$\tan^{-1}(x)+\tan^{-1}(y)+\tan^{-1}(z)=\pi$

$\Rightarrow \tan^{-1}x+\tan^{-1}y=\pi-\tan^{-1}z$

$\Rightarrow \dfrac{x+y}{1-xy}=-z \Rightarrow x+y=-z+xyz$

$\Rightarrow x+y+z=xyz$

Dividing by *xyz*, we get

$\dfrac{1}{yz}+\dfrac{1}{xz}+\dfrac{1}{xy}=1$.

Note: Students should remember this question as a formula.

Q.29 (3)

$\cos^{-1}x+\cos^{-1}(2x)=-\pi$

$\Rightarrow \cos^{-1}2x=-\pi-\cos^{-1}x$

$\Rightarrow 2x=\cos(\pi+\cos^{-1}x)$

$\Rightarrow 2x=\cos\pi(\cos\cos^{-1}x)-\sin\pi\sin(\cos^{-1}x)$

$2x=-x \Rightarrow x=0$

But x = 0 does not satisfy the given equation.

No solution will exist.

Q.30 (1) Given equation is $2\cos^{-1}x+\sin^{-1}x=\dfrac{11\pi}{6}$

$\Rightarrow \cos^{-1}x+(\cos^{-1}x+\sin^{-1}x)=\dfrac{11\pi}{6}$

$\Rightarrow \cos^{-1}x+\dfrac{\pi}{2}=\dfrac{11\pi}{6}$

$\Rightarrow \cos^{-1}x=4\pi/3$

which is not possible as $\cos^{-1}x \in [0,\pi]$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

Given = 60º + 45º = 105º

Q.2 (4)

$\therefore \quad -1 \le x \le 1$ .....(1)

$x \in R$ .....(2)

$x \le -1$ or $x \ge 1$ .....(3)

By (1) ∩ (2) ∩ (3

$\Rightarrow x \in \{-1, 1\}$

Q.3 (3)

Domain of f(x) is $x \in \{-1, 1\}$

$f(-1)=-\dfrac{\pi}{2}-\dfrac{\pi}{4}+\pi=\dfrac{\pi}{4}$

$f(1)=\dfrac{\pi}{2}+\dfrac{\pi}{4}+0=\dfrac{3\pi}{4}$

Q.4 (2)

$\cos[\tan^{-1}\{\sin(\cot^{-1}\sqrt{3})\}]=y$

$=\cos[\tan^{-1}(\sin\dfrac{\pi}{6})]=\cos\left[\tan^{-1}\dfrac{1}{2}\right]$

$=\cos\left[\cos^{-1}\dfrac{2}{\sqrt{5}}\right]=\dfrac{2}{\sqrt{5}}$

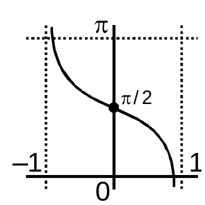
Q.5 (1)

$\sum_{i=1}^{n}\cos^{-1}\alpha_i=0$

$\cos^{-1}\alpha_1+\cos^{-1}\alpha_2+.......+\cos^{-1}\alpha_n$

so, $\cos^{-1}\alpha_1$ is always positive

So, in order to have their sum 0 all should be equal to 0



$\cos^{-1}\alpha_1=\cos^{-1}\alpha_2=.....=\cos^{-1}\alpha_n=0$

$\Rightarrow \cos^{-1}\alpha_1=0 \Rightarrow \quad \alpha_1=1$

$\cos^{-1}\alpha_2=0 \Rightarrow \quad \alpha_2=1$

⋮

⋮

$\cos^{-1}\alpha_n=0 \Rightarrow \quad \alpha_n=1$

$\therefore \sum_{i=1}^{m}\alpha_i=n$

Q.6 (2)

$\cot^{-1}\left(\frac{n}{\pi}\right) > \frac{\pi}{6}$, $n \in N$

$\frac{n}{\pi} < \cot\frac{\pi}{6} \Rightarrow n < \sqrt{3}\,\pi$

$\Rightarrow n < \sqrt{3} \times 3.14 \quad \Rightarrow n = 5$

Q.7 (4)

$\operatorname{cosec}^{-1}(\cos x)$ is define if

$\cos x \geq 1$ or $\cos x \leq -1$

$\Rightarrow \cos x = \pm 1 \Rightarrow x = n\pi$

Q.8 (4)

$\pi \leq x \leq 2\pi$ $\cos^{-1}\cos x = 2\pi - x$

Q.9 (1)

$y = \cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right)$

Let, $\cos^{-1}\frac{1}{8} = \theta \Rightarrow \cos\theta = \frac{1}{8}$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\frac{1}{8}}{2}}$

$\Rightarrow \sqrt{\frac{9}{16}} = \frac{3}{4}$

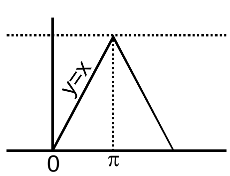
& $\cos\frac{\theta}{2} = \cos\left(\frac{\cos^{-1}\left(\frac{1}{8}\right)}{2}\right) = \frac{3}{4}$

Q.10 (4)

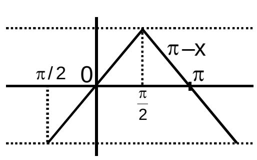
$y = \sin^{-1}[\cos\{\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\}]$

given $x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{\pi}{2} < x < \pi$

Now $\cos^{-1}(\cos x) = x$



$\sin^{-1}(\sin x) = \pi - x$



so, $y = \sin^{-1}[\cos\{x + \pi - x\}]$

$y = \sin^{-1}(\cos\pi) \Rightarrow \sin^{-1}(-1) \Rightarrow -\frac{\pi}{2}$

Q.11 (4)

$x \geq 0$, $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$

$\downarrow \quad \downarrow$

$[-1,1] \quad [-1,1] \quad x \in R$

but $x \geq 0$ so, $x \in [0, 1]$

$\theta = \frac{\pi}{2} - \tan^{-1}x$

$R_\theta : \left[\theta\big|_{x=1}, \theta\big|_{x=0}\right] = \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

Q.12 (2)

$\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2} - \cos^{-1}x - \cos^{-1}y + \frac{\pi}{2}$

$= \pi - (\cos^{-1}x + \cos^{-1}y) = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \frac{\pi}{3}$

Q.13 (2)

$\sin^{-1}x - \cos^{-1}x = \cos^{-1}\frac{\sqrt{3}}{2}$

$\frac{\pi}{2} - 2\cos^{-1}x = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{6}$

$x = \frac{\sqrt{3}}{2}$

Q.14 (2)

$\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x + \cos^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{2}$

$\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$

$\Rightarrow x = \frac{1}{\sqrt{5}}$

Q.15 (3)

$\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$

$\sin^{-1}(1) - \sin^{-1}\sqrt{\frac{3}{x}} = \frac{\pi}{6}$

$\Rightarrow \sin^{-1}\sqrt{\frac{3}{x}} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\Rightarrow \sqrt{\dfrac{3}{x}} = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad x = 4$

Q.16 (2)

By property if $x < 0$, $\tan^{-1}\dfrac{1}{x} = \cot^{-1} x - \pi$

$\therefore \quad \tan^{-1} x + \tan^{-1}\dfrac{1}{x} = \tan^{-1} x + \cot^{-1} x - \pi = \dfrac{\pi}{2} - \pi$

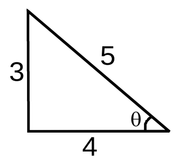
$\Rightarrow \quad \tan^{-1} x + \tan^{-1}\dfrac{1}{x} = -\dfrac{\pi}{2}$

Q.17 (4)

$y = \tan\left[\sin^{-1}\left(\dfrac{3}{5}\right) + \tan^{-1}\left(\dfrac{2}{3}\right)\right]$

Let $\theta = \sin^{-1}\dfrac{3}{5} \Rightarrow \sin\theta = \dfrac{3}{5}$

$\tan\theta = \dfrac{3}{4}$



so $\sin^{-1}\left(\dfrac{3}{5}\right) = \tan^{-1}\left(\dfrac{3}{4}\right)$

so $y = \tan\left[\tan^{-1}\left(\dfrac{3}{4}\right) + \tan^{-1}\left(\dfrac{2}{3}\right)\right]$

$$y = \frac{\tan\left\{\tan^{-1}\left(\frac{3}{4}\right)\right\} + \tan\left\{\tan^{-1}\left(\frac{2}{3}\right)\right\}}{1 - \tan\left(\tan^{-1}\left(\frac{3}{4}\right)\right).\tan\left(\tan^{-1}\left(\frac{2}{3}\right)\right)}$$

$$y = \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4}.\frac{2}{3}} = \frac{9+8}{6} = \frac{17}{6}$$

Q.18 **(3)**

$\tan\left(\dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1} x\right) + \tan\left(\dfrac{\pi}{4} - \dfrac{\cos^{-1} x}{2}\right)$

Let $\cos^{-1} x = \theta \Rightarrow x = \cos\theta$

$\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right) + \tan\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)$

$$\Rightarrow \frac{\left(1 + \tan\frac{\theta}{2}\right)^2 + \left(1 - \tan\frac{\theta}{2}\right)^2}{1 - \tan^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{2\left(\sec^2\frac{\theta}{2}\right)}{\left(1 - \tan^2\frac{\theta}{2}\right)} = \frac{2}{\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}$$

$\Rightarrow \dfrac{2}{\cos\theta} = \dfrac{2}{x}$

Q.19 (3)

$\tan^{-1} a + \tan^{-1} b = \pi + \tan^{-1}\left(\dfrac{a+b}{1-ab}\right)$

if $\quad ab > 1, a > 0, b > 0$

Q.20 (1)

$\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{3} = \tan^{-1} 1 = \dfrac{\pi}{4}$

Q.21 (4)

$\tan^{-1}\left(\dfrac{\sqrt{x^2+1} - 1}{x}\right) = \dfrac{\pi}{45^o}$

put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x, \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$\tan^{-1}\left(\dfrac{|\sec\theta| - 1}{\tan\theta}\right) = \dfrac{\pi}{45^o}$

(but $\sec\theta$ is +ve for $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$)

$\tan^{-1}\left(\dfrac{1 - \cos\theta}{\sin\theta}\right) = \dfrac{\pi}{45^o}$

$\tan^{-1}\left(\tan\dfrac{\theta}{2}\right) = \dfrac{\pi}{45^o}$

Now, $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$\left(-\dfrac{\pi}{4} < \dfrac{\theta}{2} < \dfrac{\pi}{4}\right)$

$\dfrac{\theta}{2} = \dfrac{\pi}{45^o} \Rightarrow \tan^{-1} x = 2 \times \dfrac{\pi}{45^o} = 8^o$

$x = \tan 8^o$

Q.22 (2)

$y = \cot^{-1}\left(\dfrac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right)$

given $\dfrac{\pi}{2} < x < \pi$

$$y = \cot^{-1}\left(\frac{\left|\sin\frac{x}{2} - \cos\frac{x}{2}\right| + \left|\sin\frac{x}{2} + \cos\frac{x}{2}\right|}{\left|\sin\frac{x}{2} - \cos\frac{x}{2}\right| - \left|\sin\frac{x}{2} + \cos\frac{x}{2}\right|}\right)$$

$\Rightarrow$ Now if $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Now $\sin\frac{x}{2} > \cos\frac{x}{2}$ so, modulus will open directly

$$y = \cot^{-1}\left(-\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right) = \cot^{-1}\left(-\tan\frac{x}{2}\right)$$

$$y = \pi - \cot^{-1}(\tan\frac{x}{2}) \Rightarrow \pi - \cot^{-1}\cot\left(\frac{\pi}{2} - \frac{x}{2}\right)$$

$$y = \pi - \frac{\pi}{2} + \frac{x}{2} = \frac{\pi}{2} + \frac{x}{2}$$

Q.23 (2)

$y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0 \le x \le 1$

put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$ $\theta \in [0, \pi]$

$$y = \tan^{-1}\left(\frac{1-\cos\theta}{1+\cos\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

here given, $0 \le x \le 1$

$0 \le \cos\theta \le 1$

$0 < \theta \le \frac{\pi}{2}$

comes in PVR of $\theta = \cos^{-1}x$

$0 < \frac{\theta}{2} \le \frac{\pi}{4}$

so, $y = \frac{\theta}{2} = \frac{\cos^{-1}x}{2}$

Now $y_{min} = \left.\frac{\theta}{2}\right|_{\theta=0} = 0$

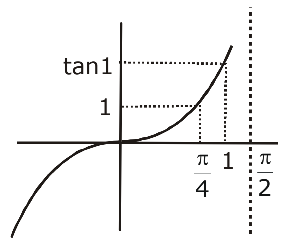
$y_{max} = \left.\frac{\theta}{2}\right|_{\theta=\frac{\pi}{2}} = \frac{\pi}{4}$

so, $\left(0, \frac{\pi}{4}\right)$

Q.24 (1)

$\tan 1 > 1$

$\tan 1 > \frac{\pi}{4}$



$\tan 1 > \tan^{-1} 1$

Q.25 (3)

$$\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$$

$$\tan^{-1}\left\{\frac{1+x+1-x}{1-(1-x^2)}\right\} = \frac{\pi}{2}$$

$$\frac{2}{x^2} = \infty \Rightarrow x^2 = 0$$

$\Rightarrow x = 0$

Q.26 (3)

$$\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$$

$$\tan^{-1}\left\{\frac{\frac{1}{2x+1} + \frac{1}{4x+1}}{1 - \frac{1}{(2x+1)(4x+1)}}\right\} = \tan^{-1}\frac{2}{x^2}$$

$$\frac{4x+1+2x+1}{(2x+1)(4x+1)-1} = \frac{2}{x^2}$$

$$\frac{6x+2}{(8x^2+6x)} = \frac{2}{x^2} \Rightarrow 6x^3 + 2x^2 = 16x^2 + 12x$$

$\Rightarrow 6x^3 - 14x^2 - 12x = 0$

$\Rightarrow x(6x^2 - 14x - 12) = 0$

$x = 0,\ 6x^2 - 14x - 12 = 0$

$x = 0,\ x = 3,\ \frac{-3}{2}$

check : $x = 0$

$\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ (Accepted)

$x = 3$

$$\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

$$\tan^{-1}\frac{\frac{1}{7} + \frac{1}{13}}{1 - \frac{1}{91}}$$

$\Rightarrow \tan^{-1}\left(\frac{13+7}{90}\right)$ (Accepted)

$x = -3/2$

$\tan^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}\left(\frac{-1}{5}\right)$

$\Rightarrow \tan^{-1}\left(\frac{-6}{9}\right)$ (Rejected)

So, 2 solutions

Q.27 (2)

$\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{5}{13}$

$= \cos^{-1}\left(\frac{3}{5}\times\frac{5}{13} - \frac{4}{5}\times\frac{12}{13}\right) = \cos^{-1}\left(-\frac{33}{65}\right)$

Q.28 (2)

$\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{3\sin 2\theta}{5+4\cos 2\theta} = 1$

$\Rightarrow \frac{6\tan\theta}{\tan^2\theta + 9} = 1$

$\Rightarrow \tan^2\theta - 6\tan\theta + 9 = 0$

$\Rightarrow (\tan\theta - 3)^2 = 0$

$\tan\theta = 3$

Q.29 (4)

$\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$= \begin{cases} 2\sin^{-1}x & \text{if } |x| \le \frac{1}{\sqrt{2}} \\ \pi - 2\sin^{-1}x & \text{if } \frac{1}{\sqrt{2}} < x \le 1 \\ -\left(\pi + 2\sin^{-1}x\right) & \text{if } -1 \le x < -\frac{1}{\sqrt{2}} \end{cases}$$

$\therefore\ 2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ is true for $|x| \le \frac{1}{\sqrt{2}}$

Q.30 (3)

$\cos^{-1}\left\{\frac{x^2}{2} + \sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right\} = \cos^{-1}\frac{x}{2} - \cos^{-1}x$

The above holds iff

$1 \ge x \ge 0$ & $1 \ge \frac{x}{2} \ge 0$

$0 \le x \le 1$ & $0 \le x \le 2$

$\downarrow \cap$

$0 \le x \le 1$

Q.31 (3)

$\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$

$\cos^{-1}(\sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p}) + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$

$\cos^{-1}0 + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$

$\cos^{-1}\sqrt{1-q} = \frac{\pi}{4} \Rightarrow 1 - q = \frac{1}{2}$

$\Rightarrow q = \frac{1}{2}$ so, $0 \le \sqrt{p} \le 1$ & $0 \le \sqrt{1-p} \le 1$

$0 \le p \le 1$ & $0 \le 1 - p \le 1$

$-1 \le -p \le 0 \Rightarrow 0 \le p \le 1$

Q.32 **(2)**

$\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\sin^{-1}\left\{\frac{x}{5}\times\frac{3}{5} + \frac{4}{5}\times\frac{\sqrt{25-x^2}}{5}\right\} = \frac{\pi}{2}$

$\frac{3x}{25} + \frac{4\sqrt{25-x^2}}{25} = 1$

$x^2 - 6x + 9 = 0 \Rightarrow (x-3)^2 = 0$

$\Rightarrow x = 3$

Q.33 (1)

$\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

$(\sin^{-1}x + \sin^{-1}(1-x)) = \cos^{-1}x$

$\sin^{-1}(x\sqrt{1-(1-x)^2} + (1-x)\sqrt{1-x^2})$

$= \sin^{-1}\sqrt{1-x^2}$

$x\sqrt{2x-x^2} = \sqrt{1-x^2}\,(1-1+x)$

$x^2(2x-x^2) - (1-x^2)x^2 = 0$

$2x^3 - x^2 = 0$

$x^2(2x-1) = 0$

$\Rightarrow x = 0, \frac{1}{2}$

Both accepted.

Q.34 (2)

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$-2\sin^{-1}x = \cos^{-1}(1-x)$

$1 - 2x^2 = 1 - x$

$2x^2 - x = 0 \Rightarrow x = 0, \frac{1}{2}$

check : $x = 0$

L.H.S. $\Rightarrow \frac{\pi}{2} - 0$

$= \frac{\pi}{2}$ = RHS

$x = \frac{1}{2}$

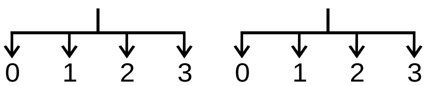
$\frac{\pi}{6} - \frac{\pi}{3} \Rightarrow -\frac{\pi}{2} \Rightarrow$ so rejected

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

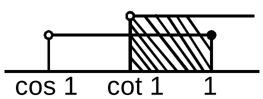
$[\cot^{-1} x] \quad + \quad [\cos^{-1} x] = 0$

$(0, \pi) \qquad\qquad [0, \pi]$



so, $0 < \cot^{-1} x < 1$ & $0 \le \cos^{-1} x < 1$

$\cot 1 < x < \infty$ & $\cos 1 < x \le 1$



so, $x \in (\cot 1, 1]$

Q.2 (A)

$([\cot^{-1}x] - 3)^2 \le 0 \Rightarrow [\cot^{-1} x] = 3$

$\Rightarrow 3 \le \cot^{-1} x < 4 \Rightarrow 3 \le \cot^{-1} x < \pi < 4$

$\Rightarrow -\infty < x \le \cot 3$

Q.3 (A)

$u = \cot^{-1} \sqrt{\tan \alpha} - \tan^{-1} \sqrt{\tan \alpha}$

then, $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = ?$

$u = \frac{\pi}{2} - 2\tan^{-1}(\sqrt{\tan\alpha})$

Now $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{4} + \tan^{-1}\sqrt{\tan\alpha}\right)$

$\Rightarrow \sqrt{\tan\alpha}$

Q.4 (D)

$x^2 + ax + \frac{\pi}{2} = 0$

$D \ge 0$

$a^2 - 4 \times \frac{\pi}{2} \times 1 \ge 0$

$a^2 - 2\pi \ge 0$

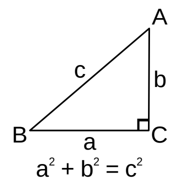
$a \ge \pm\sqrt{2\pi}$

$(-\infty, -\sqrt{2\pi}\,] \cup [\,\sqrt{2\pi}, \infty)$

Q.5 (A)

$\tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right)$ if $\angle C = 90^\circ$

$\tan^{-1}\left\{\frac{\frac{a}{b+c} + \frac{b}{c+a}}{1 - \frac{a.b}{(b+c)(c+a)}}\right\}$



$\tan^{-1}\left\{\frac{ac + a^2 + b^2 + bc}{bc + ab + c^2 - ab + ac}\right\}$

$\Rightarrow \tan^{-1}\left\{\frac{c(a+b+c)}{c(a+b+c)}\right\} \Rightarrow \frac{\pi}{4}$

JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 **(C, D)**

$x^2 - x - 2 > 0$

α satisfies it

$\Rightarrow \alpha^2 - \alpha - 2 > 0$

$(\alpha - 1)^2 - 3 > 0$

$\Rightarrow (\alpha - 1) > 3$ or $\alpha - 1 < -3$

$\alpha > 4$ or $\alpha < -2$

so C & D are correct.

Q.2 **(B,C)**

Given : $\alpha = 2\tan^{-1}(\sqrt{2} - 1)$

$\beta = 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

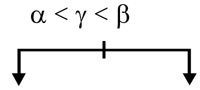
$\gamma = \cos^{-1}\left(\frac{1}{3}\right)$

$\alpha = 2 \times 22.5^\circ = 45^\circ$

$\beta = \frac{3\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{12} = 105^\circ$

$\gamma = \cos^{-1}\left(\frac{1}{3}\right) = \cos^{-1}(0.33)$

$\alpha < \gamma < \beta$



so, $\gamma > \alpha \qquad \beta > \gamma$

Q.3 **(AB)**

$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\Rightarrow$ x = y = z = 1

$\Rightarrow x^{100} + y^{100} + z^{100} - \frac{9}{x^{101}+y^{101}+z^{101}} = 0$

Q.4 **(B, C, D)**

$\cos\left[\frac{1}{2}\cos^{-1}\cos\left(-\frac{14\pi}{5}\right)\right] = \cos\left[\frac{1}{2}\cos^{-1}\cos\left(\frac{14\pi}{5}\right)\right]$

$\because \cos\theta = \cos(-\theta)$

$= \cos\left[\frac{1}{2}\left(\frac{14\pi}{5} - 2\pi\right)\right]$ since $\frac{14\pi}{5} \in (2\pi, 3\pi)$

$= \cos\left[\frac{2\pi}{5}\right]$

$= -\cos\left(\pi - \frac{2\pi}{5}\right) = -\cos\left(\frac{3\pi}{5}\right)$

$= \sin\left(\frac{\pi}{5} - \frac{2\pi}{5}\right) = \sin\left(\frac{\pi}{10}\right)$

Q.5 **(B, D)**

$6\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) = \pi$

$\Rightarrow \sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) = \frac{\pi}{6}$

$\Rightarrow x^2 - 6x + \frac{17}{2} = \frac{1}{2}$

$\Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow$ x =2, 4

Q.6 **(C, D)**

$\sin^{-1}x > \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\Rightarrow \sin^{-1} > \frac{\pi}{4} \Rightarrow x > \frac{1}{\sqrt{2}}$ also $|x| \le 1$

$\Rightarrow x \in \left(\frac{1}{\sqrt{2}}, 1\right]$

Q.7 **(A,C)**

$f(x) = \sin^{-1}x + \cos^{-1}x$

Now f(x) will be equal to $\frac{\pi}{2}$ iff arg lie b/w –1 to 1

Q.8 **(A,C,D)**

$\text{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$ $\quad |x| \ge 1$

Q.9 **(A, B, C)**

$\tan^{-1}\frac{\sqrt{1-x^2}}{1+x}$ since 0 < x < 1

$= \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$ (let $\cos^{-1}x = \theta$ $0 < \theta < \frac{\pi}{2}$)

$= \tan^{-1}\tan\frac{\theta}{2} \because \frac{\theta}{2} \in \left(0, \frac{\pi}{4}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$ ....(1)

also $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+x}{2}}$ (taking $\cos^{-1}$ on both side)

$\cos^{-1}\cos = \frac{\theta}{2} = \cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$ since $\frac{\theta}{2} \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow \frac{\theta}{2} = \cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$ ...(2)

similarly $\sin\frac{\theta}{2} = \sqrt{\frac{1-x}{2}}$

$\sin^{-1}\sin\frac{\theta}{2} = \frac{\theta}{2} = \sin^{-1}\sqrt{\frac{1-x}{2}}$

...(3)

also $\frac{\theta}{2} = \tan^{-1}\sqrt{\frac{1+x}{1-x}}$ ...(4)

Q.10 **(A,B,C)**

$\tan\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right) = \frac{a}{b}$

$\tan\left(\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right) = \frac{a}{b}$

$\tan\left(\tan^{-1}\left(\frac{17}{6}\right)\right) = \frac{a}{b}$

$\frac{a}{b} = \frac{17}{6} \Rightarrow a - b = 11$

$\Rightarrow$ a + b = 23

$\Rightarrow$ 3b = 3.6 = 18 = a + 11

Q.11 **(A, D)**

$\sum_{n=1}^{\infty} \tan^{-1}\frac{4n}{n^4 - 2n^2 + 2}$

$= \lim_{k\to\infty}\sum_{n=1}^{k}\left\{\tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2\right\}$

$= \lim_{k\to\infty}\left\{\tan^{-1}(k+1)^2 + \tan^{-1}k^2 - \tan^{-1}1 - \tan^{-1}0\right\}$

$= \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{4} - 0 = \frac{3\pi}{4}$

Also $\tan^{-1}2 + \tan^{-1}3 = \pi + \tan^{-1}\left(\frac{3+2}{1-3.2}\right)$

Since xy = 6 > 1

$= \frac{3\pi}{4}$

and $\sec^{-1}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$

Q.12 **(B,C)**

$2x = \tan(2\tan^{-1} a) + 2\tan(\tan^{-1} a + \tan^{-1} a^3)$

$2x = \tan(\tan^{-1} a + \tan^{-1} a) + 2\tan(\tan^{-1} a + \tan^{-1} a^3)$

$2x = \frac{2a}{1-a^2} + 2\frac{a+a^3}{(1-a^2)(1+a^2)}$

$x = \frac{a}{1-a^2} + \frac{a}{(1-a^2)}$

$a^2x - x + 2a = 0$ (A) is valid

& $a \neq -1$ & 1 (D) is valid

Q.13 (**B,D**)

$6\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) = \pi$

$\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) = \frac{\pi}{6}$

$x^2 - 6x + \frac{17}{2} - \frac{1}{2} = 0$

x = 2, 4

Q.14 (**A,C**)

Given $a = \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$

$b = \tan^{-1}\left(-\sqrt{3}\right) - \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$a = -\frac{\pi}{4} + \pi - \frac{\pi}{3} \Rightarrow \frac{5\pi}{12}$

$b = -\frac{\pi}{3} - \pi + \frac{\pi}{3} \Rightarrow -\pi$

$a - b = \frac{17\pi}{12}$ & $a + b = -\frac{7\pi}{12}$

Q.15 (**A, C)**

$\cos^{-1} x = \tan^{-1} x \Rightarrow x \in (0, 1]$

$\tan^{-1}\frac{\sqrt{1-x^2}}{x} = \tan^{-1} x$

taking tan on both side $\sqrt{1-x^2} = x^2$

$\Rightarrow 1 - x^2 = x^4 \Rightarrow x^4 + x^2 - 1 = 0$

$x^2 = \frac{-1+\sqrt{5}}{2}$ since x^2 is +ve avoid negative result

$\sin(\cos^{-1} x) = \sin(\sin^{-1}\sqrt{1-x^2}) = \sqrt{1-x^2} = x^2$

$= \frac{\sqrt{5}-1}{2}$

$\tan(\cos^{-1} x) = \tan(\tan^{-1} x) = x \neq \frac{\sqrt{5}-1}{2}$

Comprehension # 1 (Q. No. 16 to 18)

Q.16 (D)

Q.17 (C)

Q.18 (**D)**

Let $\cos^{-1} x = \theta$, then $x = \cos\theta$ and $0 \le \theta \le \pi$

$\therefore \sin^{-1}\sqrt{1-x^2} = \sin^{-1}(\sin\theta)$

$= \begin{cases} \theta & \text{if } 0 \le \theta \le \frac{\pi}{2} \\ \pi - \theta & \text{if } \frac{\pi}{2} < \theta \le \pi \end{cases}$

$= \begin{cases} \cos^{-1} x & \text{if } 0 \le x \le 1 \\ \pi - \cos^{-1} x & \text{if } -1 \le x < 0 \end{cases}$

$\therefore \cos^{-1} x = \sin^{-1}\sqrt{1-x^2}$ if $0 < x < 1$ is true.

Sol.17 Let $\sin^{-1} x = \theta$, then $x = \sin\theta$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\therefore \cos^{-1}\sqrt{1-x^2} = \cos^{-1}(\cos\theta)$

$= \begin{cases} -\theta, & -\frac{\pi}{2} \le \theta \le 0 \\ \theta, & 0 \le \theta \le \frac{\pi}{2} \end{cases}$

$= \begin{cases} -\sin^{-1} x, & -1 \le x \le 0 \\ \sin^{-1} x, & 0 \le x \le 1 \end{cases}$

$\therefore \sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$ if $0 < x < 1$ is true

Sol.18 Let $\cos^{-1} x = \theta$, then $x = \cos\theta$ and $0 \le \theta \le \pi$

$\therefore \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \tan^{-1}(\tan\theta)$

$$= \begin{cases} \theta & , \ 0 \le \theta < \frac{\pi}{2} \\ \theta - \pi & , \ \frac{\pi}{2} < \theta \le \pi \end{cases}$$

$$= \begin{cases} \cos^{-1} x & , \ 0 < x \le 1 \\ -\pi + \cos^{-1} x & , \ -1 \le x < 0 \end{cases}$$

i.e $\cos^{-1} x = \pi + \tan^{-1} \frac{\sqrt{1-x^2}}{x}$, $-1 < x < 0$ is correct.

Comprehension # 2 (Q. No. 19 to 21)

Q.19 (A)
Q.20 (C)
Q.21 (**B**)

Sol.19 $\sin^{-1} x < \frac{3\pi}{4}$

$x > \sin \frac{3\pi}{4}$

$1 \ge x > \frac{1}{\sqrt{2}}$

$x \in \left(\frac{1}{\sqrt{2}}, 1\right]$

Sol.20 $\sin^{-1}(-1) + \text{cosec}^{-1}(-1)$

$= \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

Sol.21 If $-1 \le x \le 1$
then $-1 \le -x \le 1$

So $\frac{3\pi}{4} \le \tan^{-1}(-x) \le \frac{5\pi}{4}$

Q.22 (A) → (t), (B) → (s), (C) → (t), (D) → (p)

(A) Given expression = $2\sin^{-1}x - \frac{\pi}{2}$ maximum at

(B)Given expression = $t^2 + \left(\frac{\pi}{2} - t\right)^2$, $t = \sin^{-1} x$

minimum at $t = \frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}}$

(C) Similar to (B)

(D) Given expression = $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ =

$\left(\frac{\pi}{2}\right)^3 - 3\sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x\right).\frac{\pi}{2}$

$= \frac{3\pi}{2}(\sin^{-1}x)^2 - \frac{3\pi^2}{4}\sin^{-1}x + \frac{\pi^3}{8}$

This is quadratic in $\sin^{-1}x$. Therefore it will give maximum value when $\sin^{-1}x = -\frac{\pi}{2} \Rightarrow x = -1$

Let the domain and range of inverse circular functions are defined as follows Domain Range

$\sin^{-1}x$ $[-1, 1]$ $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ $\cos^{-1}x$ $[-1, 1]$ $[0, \pi]$ $\tan^{-1}x$ R $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$\cot^{-1}x$ R $(0, \pi)$ $\text{cosec}^{-1}x$ $(-\infty, -1] \cup [1, \infty)$ $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$

$\sec^{-1}x$ $(-\infty, -1] \cup [1, \infty)$ $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Q.23 (A) → (p), (B) → (q), (C) → (r) (D) → (s)

(A) G.E. = $\dfrac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\frac{2}{3}} = \frac{17}{6}$

(B) G.E. $\dfrac{\dfrac{2.\frac{1}{5}}{1-\frac{1}{25}} - 1}{1 + \dfrac{2.\frac{1}{5}}{1-\frac{1}{25}}.1} = \frac{-7}{17}$

(C) G.E. = $\sqrt{\dfrac{1+\frac{1}{8}}{2}}$ =

(D) G.E. = $\cos \tan^{-1} \frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{3}$

Q.24 (A) → (p, q, r, s); (B) → (p); (C) → (P,R,S); (D) → (p)

(A) $f(x) = \sin^{-1}\left(\frac{2}{|\sin x - 1| + |\sin x + 1|}\right)$

For all values of x
| sin x – 1| will open negative
|sin x + 1| is positive

$f(x) = \sin^{-1}(1)$ $\frac{\pi}{2}$

so P, Q, R. S

(B) $f(x) = \cos^{-1}(|x-1| - |x-2|)$
In $(-\infty, 1]$ $-x + 1 + x - 2 = -1$
$\cos^{-1}(-1) = \pi$

(1, 2) not in domain.
[2, ∞) $\cos^{-1}[1] = 0$
So, Ans. P

(C) $f(x) = \sin^{-1}\left[\frac{\pi}{\left|\sin^{-1}x - \frac{\pi}{2}\right| + \left|\sin^{-1}x + \frac{\pi}{2}\right|}\right]$

Domain $|x| \le 1$
Ans. P,R,S
(D) $f(x) = \cos(\cos^{-1}|x|)$
$+ \sin^{-1}(\sin x) - \text{cosec}^{-1}(\text{cosec } x) + \text{cosec}^{-1}|x|$
domain of f(x)
$|x| = 1$
$x = \pm 1$

NUMERICAL VALUE BASED

Q.1 (30)

Case-I $x \ge 0$
Let $\cot^{-1} x = \theta$

$\therefore \quad \theta \in \left(0, \frac{\pi}{2}\right]$

$\Rightarrow \quad x = \cot\theta$

$\therefore \quad \sin\theta = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \quad \sin^{-1}\sin\theta = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \quad \theta = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

Case-II
$x < 0$
Let $\cot^{-1} x = \theta$

$\therefore \quad \theta \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \quad \cot\theta = x$

$\therefore \quad \sin\theta = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \quad \sin^{-1}\sin\theta = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \quad \pi - \theta = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \quad \theta = \pi - \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

Therefore

LHS

$$= \begin{cases} \cos\tan^{-1}\sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}, & \text{if } x \ge 0 \\ \cos\tan^{-1}\sin\left(\pi - \sin^{-1}\frac{1}{\sqrt{1+x^2}}\right), & \text{if } x < 0 \end{cases} = \cos\tan^{-1}$$

$\sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}$; $x \in R = \cos\tan^{-1}\frac{1}{\sqrt{1+x^2}}$

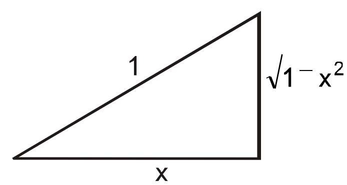
Let $\quad \phi = \tan^{-1}\frac{1}{\sqrt{1+x^2}}$

As $\frac{1}{\sqrt{1+x^2}} \in (0, 1] \quad \therefore \ \phi \in \left(0, \frac{\pi}{4}\right]$

$\therefore \ \tan\phi = \frac{1}{\sqrt{1+x^2}} \quad \therefore \ \cos\phi = \sqrt{\frac{1+x^2}{2+x^2}}$

Q.2 (54)

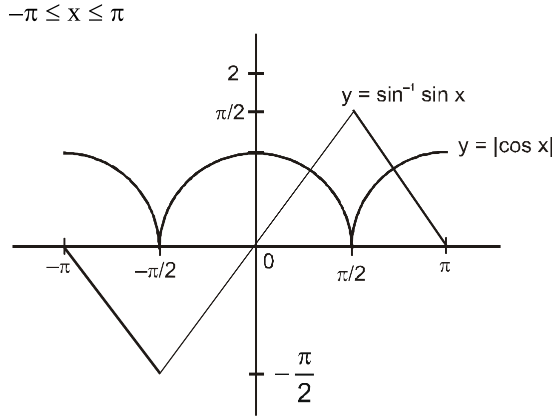
$\sec\tan^{-1}\left(\frac{2\sqrt{1-x^2}}{2x}\right) = \sec\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \frac{1}{x}$



$\sum_{r=2}^{10} f\left(\frac{1}{r}\right) = 2 + 3 + \ldots\ldots + 10 = 54$

Q.3 (20)

Given equation is $|\cos x| = \sin^{-1}(\sin x)$
$-\pi \le x \le \pi$



Number of solution = 2

KVPY
PREVIOUS YEAR'S

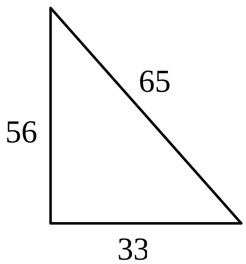
Q.1 (A)

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$$= \text{cosec}\left(\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right)$$

$$= \text{cosec}\left(\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right)$$

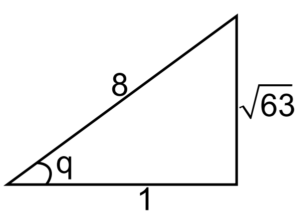


$$= \text{cosectan}\left(\tan^{-1}\left(\frac{56}{33}\right)\right)$$

$$= \frac{65}{56}$$

Q.2 (1)

Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta \qquad \Rightarrow \sin\theta = \frac{\sqrt{63}}{8}$



$$\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \tan\left(\frac{\theta}{4}\right) = \frac{1-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= \frac{1-\sqrt{\frac{1+\cos\theta}{2}}}{\sqrt{\frac{1-\cos\theta}{2}}} = \frac{1-\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{1}{\sqrt{7}}$$

Q.3 (3)

Let $\sin^{-1}x = a\lambda$, $\cos^{-1}x = b\lambda$, $\tan^{-1}y = c\lambda$

$$\Rightarrow (a+b)\lambda = \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{a+b} = 2\lambda$$

Now $\cos\left(\frac{\pi}{a+b}\right) = \cos(2\lambda c) = \cos(2\tan^{-1}y)$

$$= \frac{1-y^2}{1+y^2}$$

Q.4 (3)

$$\tan^{-1}\left(\frac{a+b}{1-ab}\right) = \frac{\pi}{4}$$

$\Rightarrow$ a+b=1–ab

$\Rightarrow$ (1+a)(1+b)=2

Now, $a+b-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)....\infty$

$$= \left(a-\frac{a^2}{2}+\frac{a^3}{3}....\right)+\left(b-\frac{b^2}{2}+\frac{b^3}{3}.....\right)$$

$$= \ell n(1+a)+\ell n(1+b) = \ell n(1+a)(1+b) = \ell n2$$

Q.5 01.00

$$\tan\left(\lim_{x\to\infty}\sum_{r=1}^{n}[\tan^{-1}(r+1)-\tan^{-1}(r)]\right)$$

$$= \tan\left(\lim_{x\to\infty}\left(\tan^{-1}(n+1)-\tan^{-1}\frac{\pi}{4}\right)\right)$$

$$= \tan\left(\frac{\pi}{4}\right) = 1$$

Q.6 **(3)**

$$S_k = \sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$$

Divide by 3^{2r}

$$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r}.2+3}\right)$$

$$\sum_{r=1}^{k}\tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$$

Let $\left(\frac{2}{3}\right)^r = t$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{t-\frac{2t}{3}}{1+t.\frac{2t}{3}}\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}(t)-\tan^{-1}\left(\frac{2t}{3}\right)\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}\left(\frac{2}{3}\right)^{r}-\tan^{-1}\left(\frac{2}{3}\right)^{r+1}\right)$$

$$S_k = \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}$$

$$S_\infty = \lim_{k\to\infty}\left(\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}\right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}(0)$$

$$\therefore\quad S_\infty = \tan^{-1}\left(\frac{2}{3}\right)=\cot^{-1}\left(\frac{3}{2}\right)$$

Q.7 **(3)**

$$\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$$

$$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right)=\sin^{-1}x$$

$$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x$$

$x = 0,\ 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$ squaring we get

$16(25-9x^2)$

$= 625+9(25-16x^2)-150\sqrt{25-16x^2}$

$400 = 625 + 225 - 150\sqrt{25-16x^2}$

$\sqrt{25-16x^2} = 3 \Rightarrow 25 - 16x^2 = 9$

$\Rightarrow x^2 = 1$

Put x = 0, 1, −1 in the original equation

We see that all values satisfy the original equation.

Number of solution = 3

Q.8 **(1)**

$\cot^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18)+.....$

$$= \sum_{n=1}^{100}\tan^{-1}\left(\frac{2}{4n^2}\right)$$

$$= \sum_{n=1}^{100}\tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$= \sum_{n=1}^{100}\tan^{-1}(2n\quad 1)\quad \tan^{-1}(2n\quad 1)$$

$= \tan^{-1}201 - \tan^{-1}1$

$$= \tan^{-1}\left(\frac{200}{202}\right)$$

$$\therefore\ \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$$

$\alpha = 1.01$

Q.9 **(1)**

$$\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\frac{8}{31}$$

Taking tangent both sides :-

$$\frac{(x+1)+(x-1)}{1-(x^2-1)}=\frac{8}{31}$$

$$\Rightarrow \frac{2x}{2-x^2}\quad\frac{8}{31}$$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow\ x = -8, \frac{1}{4}$

But, if $x = \frac{1}{4}$

$$\tan^{-1}(x+1)\in\left(0,\frac{\pi}{2}\right)$$

$$\&\ \cot^{-1}\left(\frac{1}{x-1}\right)\in\left(\frac{\pi}{2},\pi\right)$$

$\Rightarrow$ LHS > $\frac{\pi}{2}$ & RHS < $\frac{\pi}{2}$ (Not possible)

Hence, x = – 8

Q.10 **(2)**

Given equation

$$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$$

Now, $\sin^{-1}\left[x^2+\frac{1}{3}\right]$ is defined if

$-1 \le x^2+\frac{1}{3} < 2 \Rightarrow \frac{-4}{3} \le x^2 < \frac{5}{3}$

$\Rightarrow \boxed{0 \le x^2 < \frac{5}{3}}$ ....(1)

and $\cos^{-1}\left[x^2-\frac{2}{3}\right]$ is defined if

$-1 \le x^2-\frac{2}{3} < 2 \Rightarrow \frac{-1}{3} \le x^2 < \frac{8}{3}$

$\Rightarrow \boxed{0 \le x^2 < \frac{8}{3}}$ ....(2)

So, form (1) and (2) we can conclude

$\boxed{0 \le x^2 < \frac{5}{3}}$

Case - I if $0 \le x^2 < \frac{2}{3}$

$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$

$\Rightarrow x + \pi = x^2$

$\Rightarrow x2 = \pi$

but $\pi \notin \left[0, \frac{2}{3}\right)$

$\Rightarrow$ No value of 'x'

Case - II if $\frac{2}{3} \le x^2 < \frac{5}{3}$

$\sin^{-1}(1) + \cos^{-1}(0) = x^2$

$\Rightarrow \frac{\pi}{2}+\frac{\pi}{2} = x^2$

$\Rightarrow x2 = \pi$

but $\pi \notin \left[\frac{2}{3}, \frac{5}{3}\right)$

$\Rightarrow$ No value of 'x'

So, number of solutions of the equation is zero.

Q.11 **(4)**

Q.12 (2)

Q.13 (3)

$\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$

$\Rightarrow (2\pi - 5) + (6 - 2\pi) - (12 - 4\pi)$

$\Rightarrow 4\pi - 11.$

Q.14 **(3)**

Q.15 (4)

Q.16 (2)

Q.17 (3)

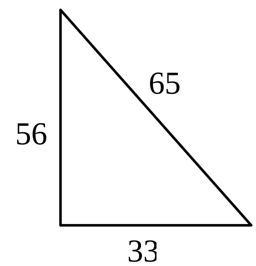
Q.18 (3)

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (1)

$= \text{cosec}\left(\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right)$

$= \text{cosec}\left(\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right)$



$= \text{cosectan}\left(\tan^{-1}\left(\frac{56}{33}\right)\right)$

$= \frac{65}{56}$

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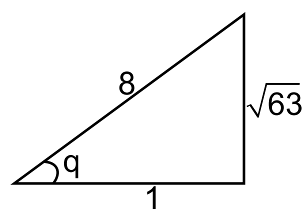
Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta \qquad \Rightarrow \sin\theta = \frac{\sqrt{63}}{8}$



$\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \tan\left(\frac{\theta}{4}\right) = \frac{1-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$= \frac{1-\sqrt{\frac{1+\cos\theta}{2}}}{\sqrt{\frac{1-\cos\theta}{2}}} = \frac{1-\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{1}{\sqrt{7}}$

Q.3 (3)

Sol. Let $\sin^{-1}x = a\lambda$, $\cos^{-1}x = b\lambda$, $\tan^{-1}y = c\lambda$

$\Rightarrow (a+b)\lambda = \frac{\pi}{2}$

$\Rightarrow \dfrac{\pi}{a+b} = 2\lambda$

Now $\cos\left(\dfrac{\pi}{a+b}\right) = \cos(2\lambda c) = \cos(2\tan^{-1}y)$

$= \dfrac{1-y^2}{1+y^2}$

Q.4 (3)

$\tan^{-1}\left(\dfrac{a+b}{1-ab}\right) = \dfrac{\pi}{4}$

$\Rightarrow a+b=1-ab$

$\Rightarrow (1+a)(1+b)=2$

Now, $a+b-\left(\dfrac{a^2+b^2}{2}\right)+\left(\dfrac{a^3+b^3}{3}\right)....\infty$

$=\left(a-\dfrac{a^2}{2}+\dfrac{a^3}{3}....\right)+\left(b-\dfrac{b^2}{2}+\dfrac{b^3}{3}.....\right)$

$=\ell n(1+a)+\ell n(1+b)=\ell n(1+a)(1+b)=\ell n2$

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$\tan\left(\lim\limits_{x\to\infty}\sum\limits_{r=1}^{n}[\tan^{-1}(r+1)-\tan^{-1}(r)]\right)$

$= \tan\left(\lim\limits_{x\to\infty}\left(\tan^{-1}(n+1)-\tan^{-1}\dfrac{\pi}{4}\right)\right)$

$= \tan\left(\dfrac{\pi}{4}\right) = 1$

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$S_k = \sum\limits_{r=1}^{k}\tan^{-1}\left(\dfrac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

Divide by 3^{2r}

$\sum\limits_{r=1}^{k}\tan^{-1}\left(\dfrac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r}.2+3}\right)$

$\sum\limits_{r=1}^{k}\tan^{-1}\left(\dfrac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$

Let $\left(\dfrac{2}{3}\right)^r = t$

$\sum\limits_{r=1}^{k}\tan^{-1}\left(\dfrac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$

$\sum\limits_{r=1}^{k}\tan^{-1}\left(\dfrac{t-\frac{2t}{3}}{1+t.\frac{2t}{3}}\right)$

$\sum\limits_{r=1}^{k}\left(\tan^{-1}(t)-\tan^{-1}\left(\dfrac{2t}{3}\right)\right)$

$\sum\limits_{r=1}^{k}\left(\tan^{-1}\left(\dfrac{2}{3}\right)^r-\tan^{-1}\left(\dfrac{2}{3}\right)^{r+1}\right)$

$S_k = \tan^{-1}\left(\dfrac{2}{3}\right)-\tan^{-1}\left(\dfrac{2}{3}\right)^{k+1}$

$S_\infty = \lim\limits_{k\to\infty}\left(\tan^{-1}\left(\dfrac{2}{3}\right)-\tan^{-1}\left(\dfrac{2}{3}\right)^{k+1}\right)$

$= \tan^{-1}\left(\dfrac{2}{3}\right)-\tan^{-1}(0)$

$\therefore\ S_\infty = \tan^{-1}\left(\dfrac{2}{3}\right) = \cot^{-1}\left(\dfrac{3}{2}\right)$

Q.7 **(3)**

$\sin^{-1}\dfrac{3x}{5}+\sin^{-1}\dfrac{4x}{5}=\sin^{-1}x$

$\sin^{-1}\left(\dfrac{3x}{5}\sqrt{1-\dfrac{16x^2}{25}}+\dfrac{4x}{5}\sqrt{1-\dfrac{9x^2}{25}}\right)=\sin^{-1}x$

$\dfrac{3x}{5}\sqrt{1-\dfrac{16x^2}{25}}+\dfrac{4x}{5}\sqrt{1-\dfrac{9x^2}{25}}=x$

$x = 0,\ 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$ squaring we get $16(25-9x^2)$

$=625+9(25-16x^2)-150\sqrt{25-16x^2}$

$400 = 625+225-150\sqrt{25-16x^2}$

$\sqrt{25-16x^2}=3 \Rightarrow 25-16x^2=9$

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Put x = 0, 1, −1 in the original equation

We see that all values satisfy the original equation.

Number of solution = 3

Q.8 **(1)**

$Cot^{-1}(\alpha) = cot^{-1}(2) + cot^{-1}(8) + cot^{-1}(18) + ...$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$= \sum_{n=1}^{100} \tan^{-1}(2n \quad 1) \quad \tan^{-1}(2n \quad 1)$

$= \tan^{-1} 201 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{200}{202}\right)$

$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$

$\alpha = 1.01$

Q.9 **(1)**

$\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\frac{8}{31}$

Taking tangent both sides :-

$\frac{(x+1)+(x-1)}{1-(x^2-1)} = \frac{8}{31}$

$\Rightarrow \frac{2x}{2-x^2} \quad \frac{8}{31}$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow x = -8, \frac{1}{4}$

But, if $x = \frac{1}{4}$

$\tan^{-1}(x+1) \in \left(0, \frac{\pi}{2}\right)$

& $\cot^{-1}\left(\frac{1}{x-1}\right) \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow$ LHS $> \frac{\pi}{2}$ & RHS $< \frac{\pi}{2}$

(Not possible)

Hence, $x = -8$

Q.10 **(2)**

Given equation

$\sin^{-1}\left[x^2+\frac{1}{3}\right] + \cos^{-1}\left[x^2-\frac{2}{3}\right] = x^2$

Now, $\sin^{-1}\left[x^2+\frac{1}{3}\right]$ is defined if

$-1 \le x^2 + \frac{1}{3} < 2 \Rightarrow \frac{-4}{3} \le x^2 < \frac{5}{3}$

$\Rightarrow \boxed{0 \le x^2 < \frac{5}{3}}$ ....(1)

and $\cos^{-1}\left[x^2-\frac{2}{3}\right]$ is defined if

$-1 \le x^2 - \frac{2}{3} < 2 \Rightarrow \frac{-1}{3} \le x^2 < \frac{8}{3}$

$\Rightarrow \boxed{0 \le x^2 < \frac{8}{3}}$ ....(2)

So, form (1) and (2) we can conclude

$\boxed{0 \le x^2 < \frac{5}{3}}$

Case - I if $0 \le x^2 < \frac{2}{3}$

$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$

$\Rightarrow x + \pi = x^2$

$\Rightarrow x2 = \pi$

but $\pi \notin \left[0, \frac{2}{3}\right)$

$\Rightarrow$ No value of 'x'

Case - II if $\frac{2}{3} \le x^2 < \frac{5}{3}$

$\sin^{-1}(1) + \cos^{-1}(0) = x^2$

$\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = x^2$

$\Rightarrow x2 = \pi$

but $\pi \notin \left[\frac{2}{3}, \frac{5}{3}\right)$

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So, number of solutions of the equation is zero.

Q.11 **(4)**

Q.12 (2)

Q.13 (3)

$\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$

$\Rightarrow (2\pi - 5) + (6 - 2\pi) - (12 - 4\pi)$

$\Rightarrow 4\pi - 11.$

Q.14 **(3)**

Q.15 (4)

Q.16 (2)

Q.17 (3)

Q.18 (3)

EXERCISES

ELEMENTRY

Q.1 (4)

Since $\lim_{x\to0-}\frac{|x|}{x}=-1$ and $\lim_{x\to0+}\frac{|x|}{x}=1$, hence limit does not exist.

Q.2 (1)

$\lim_{x\to n+0}(x-[n])=\lim_{x\to n+0}x-\lim_{x\to n+0}[n]=n-n=0$.

Q.3 (1)

$$\lim_{x\to1}\frac{\log[(x-1)+1]}{x-1}=1.$$

Aliter : Apply L-Hospital's rule,

$$\lim_{x\to1}\frac{\log x}{x-1}=\lim_{x\to1}\frac{\frac{1}{x}}{1}=1$$

Q.4 (2)

$$\lim_{x\to2}\frac{x^n-2^n}{x-2}=n.2^{n-1}$$

$\Rightarrow n.2^{n-1}=80$

$\Rightarrow n=5$

Q.5 (1)

$$\lim_{x\to0}\frac{x.2\sin^2x}{x^2}=2.\lim_{x\to0}\left(\frac{\sin x}{x}\right)^2\cdot\lim_{x\to0}x=0.$$

Q.6 (3)

$\lim_{x\to0}kx\,\text{cosec}\,x=\lim_{x\to0}x\,\text{cosec}\,kx$

$\Rightarrow k.\lim_{x\to0}\frac{x}{\sin x}=\frac{1}{k}\lim_{x\to0}\frac{kx}{\sin kx}$

$\Rightarrow k=\frac{1}{k}\Rightarrow k=\pm1$.

Q.7 (3)

$$\lim_{x\to0}\frac{x^3\cot x}{1-\cos x}=\lim_{x\to0}\left(\frac{x^3\cot x}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}\right)$$

$$=\lim_{x\to0}\left(\frac{x}{\sin x}\right)^3\times\lim_{x\to0}\cos x\times\lim_{x\to0}(1+\cos x)=2$$

Q.8 (4)

$$\lim_{x\to0}\frac{x(e^x-1)}{1-\cos x}=\lim_{x\to0}\frac{2x(e^x-1)}{4.\sin^2\frac{x}{2}}$$

$$=2\lim_{x\to0}\left[\frac{(x/2)^2}{\sin^2\frac{x}{2}}\right]\left(\frac{e^x-1}{x}\right)=2.$$

Q.9 (4)

$$\lim_{x\to0}\frac{2\sin4x\cos2x}{2\sin x\cos4x}=\lim_{x\to0}4\left(\frac{\sin4x}{4x}\right)\left(\frac{x}{\sin x}\right)\frac{\cos2x}{\cos4x}=4$$

Aliter : $$\lim_{x\to0}\frac{\frac{2\sin2x}{2x}+\frac{6\sin6x}{6x}}{\frac{5\sin5x}{5x}-\frac{3\sin3x}{3x}}=\frac{2+6}{5-3}=4.$$

Q.10 (1)

Expand log(1 + x) and then solve.

Aliter : Apply L-Hospital's rule,

$$\lim_{x\to0}\left[\frac{x-\log(1+x)}{x^2}\right]$$

$$=\lim_{x\to0}\frac{1-\frac{1}{1+x}}{2x}=\lim_{x\to0}\frac{1}{2}\left(\frac{1}{1+x}\right)^2=\frac{1}{2}.$$

Q.11 (2)

Apply L-Hospital's rule,

$$\lim_{x\to0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$$

$$=\lim_{x\to0}\frac{\cos x}{2\sqrt{1+\sin x}}+\frac{\cos x}{2\sqrt{1-\sin x}}=\frac{1}{2}+\frac{1}{2}=1.$$

Q.12 (1)

$$\lim_{\theta\to\pi/2}\frac{1-\sin\theta}{\cos\theta}=\lim_{\theta\to\pi/2}\frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}=0$$

Q.13 (1)

Apply the L-Hospital's rule, $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$.

Q.14 (3)

$\frac{d}{da}[a^2\sin a]=2a\sin a+a^2\cos a.$

Aliter : Apply L-Hospital's rule,

$$\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{2(a+h)\sin(a+h)+(a+h)^2\cos(a+h)}{1}$$

$$=2a\sin a+a^2\cos a.$$

Q.15 (1).

$$\lim_{x\to 3}\left\{\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\right\}=\lim_{x\to 3}\frac{(x-3)\left\{\sqrt{x-2}+\sqrt{4-x}\right\}}{2(x-3)}=1$$

Aliter : Apply *L*-Hospital's rule.

Q.16 (2) $\lim_{x\to\pi/4}\frac{(\sqrt{2}-\sec x)\cos x(1+\cot x)}{\cot x[2-\sec^2 x]}$

$$=\lim_{x\to\pi/4}\frac{\sin x(1+\cot x)}{(\sqrt{2}+\sec x)}=\frac{\frac{1}{\sqrt{2}}(2)}{\sqrt{2}+\sqrt{2}}=\frac{1}{2}.$$

Aliter : Apply L-Hospital's rule.

Q.17 (4)

$$\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}=\lim_{x\to 0}\frac{|\sin x|}{x}$$

So, $\lim_{x\to 0+}\frac{|\sin x|}{x}=1$ and $\lim_{x\to 0-}\frac{|\sin x|}{x}=-1$

Hence limit does not exist.

Q.18 (1)

Let $y=\lim_{x\to\infty}\left(1+\frac{1}{mx}\right)^x=\lim_{x\to\infty}\left(1+\frac{1}{mx}\right)^{mx\cdot\frac{1}{m}}$

$\Rightarrow y=e^{1/m},\ \left(\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e\right).$

Q.19 (1)

$\lim_{x\to 2}\frac{x^3-8}{x^2-4},\qquad \left(\frac{0}{0}\text{form}\right)$

Applying L-Hospital's rule, we get

$$\lim_{x\to 2}\frac{3x^2}{2x}=\lim_{x\to 2}\frac{3\times 2\times 2}{2\times 2}=3$$

Q.20 (4)

Let $y=\lim_{x\to 3}\frac{x^3-x^2-18}{x-3},\ \left(\frac{0}{0}\text{form}\right)$

Applying L-Hospital's rule, we get

$y=\lim_{x\to 3}3x^2-2x=(27-6)=21$.

Q.21 (1)

$$\lim_{x\to\infty}\frac{2x^2+3x+4}{3x^2+3x+4}=\lim_{x\to\infty}\frac{2+\frac{3}{x}+\frac{4}{x^2}}{3+\frac{3}{x}+\frac{4}{x^2}}=\frac{2}{3}$$

Q.22 (3)

$$\lim_{x\to 2}\left(\frac{3^{x/2}-3}{3^x-9}\right)=\lim_{x\to 2}\left(\frac{3^{x/2}-3}{(3^{x/2})^2-3^2}\right)$$

$$=\lim_{x\to 2}\frac{1}{3^{x/2}+3}=\frac{1}{6}.$$

Q.23 (1)

$$\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]}=\lim_{x\to\infty}\frac{\log x^n}{[x]}-\lim_{x\to\infty}\frac{[x]}{[x]}=0-1=-1.$$

Q.24 (3)

$$\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$$

By L-Hospital's rule, $\lim_{x\to 0}\frac{\frac{1}{3+x}+\frac{1}{3-x}}{1}=k\ \Rightarrow$

$\frac{2}{3}=k$

Q.25 (3)

Using L-Hospital's rule,

$$\lim_{\theta\to\frac{\pi}{2}}\frac{-1}{-\text{cosec}^2\theta}=1.$$

Q.26 (1)

$$\lim_{n\to\infty}\frac{1-n^2}{\Sigma n}$$

$$=\lim_{n\to\infty}\frac{(1-n)(1+n)}{\frac{1}{2}n(n+1)}=\lim_{n\to\infty}\frac{2(1-n)}{n}$$

$= \lim_{n\to\infty} 2\left(\frac{1}{n}-1\right) = 2(0-1) = -2$.

Q.27 (4)

$\lim_{x\to 0} \frac{\log_e(1+x)}{3^x-1}$, $\left(\frac{0}{0} \text{ form}\right)$

Using L-Hospital's rule,

$\lim_{x\to 0} \frac{\frac{1}{1+x}}{3^x \log_e 3} = \frac{1}{\log_e 3} = \log_3 e$.

Q.28 (3)

$\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b} = \lim_{x\to\infty}\left(1+\frac{a-b}{x+b}\right)^{x+b}$

$= \lim_{x\to\infty}\left\{\left(1+\frac{a-b}{x+b}\right)^{\frac{x+b}{a-b}}\right\}^{a-b} = e^{a-b}$

Q.29 (2)

$\lim_{x\to a} \frac{f(a)[g(x)-g(a)]-g(a)[f(x)-f(a)]}{[x-a]}$

$= f(a)g'(a) - g(a)f'(a) = 2\times 2-(-1)(1) = 5.$

Q.30 (2)

$\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+.....+\frac{n}{1-n^2}\right]$

$= \lim_{n\to\infty}\frac{\Sigma n}{1-n^2} = \frac{1}{2}\lim_{n\to\infty}\frac{n^2+n}{1-n^2} = -\frac{1}{2}$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (4)

$\ell im_{x\to 0^+} \sec x > 1$

So ℓimit not exist

Q.2 **(3)**

$\ell im_{x\to 1} (1 - x + [x-1] + [1-x])$

L.H.L.= $\ell im_{x\to 1^-} (1 - x + [x-1] + [1-x])$

$= \ell im_{h\to 0} (1 - (1-h) + [1-h-1] + [1-1+h])$

$= \ell im_{h\to 0} (h + [-h] + [h])$

$= 0 - 1 + 0 = -1$

R.H.L. = $\ell im_{x\to 1^+} (1 - x + [x-1] + [1-x])$

$= \ell im_{h\to 0} (1 - (1+h) + [1+h-1] + [1-(1+h)]$

$= \ell im_{h\to 0} (-h + [h] + [-h])$

$= 0 + 0 - 1 = -1$

L.H.L. = R.H.L. = – 1

so $\ell im_{x\to 1} (1 - x + [x-1] + [1-x]) = -1$

Q.3 **(4)**

$\lim_{x\to\infty} \sec^{-1}\left(\frac{x}{x+1}\right)$ Put $x = \frac{1}{y}$

$= \lim_{y\to 0} \sec^{-1}\left(\frac{1}{y+1}\right) = \lim_{y\to 0} \cos^{-1}(y+1)$ & $y = 0^+$

$\lim_{y\to 0^+} \cos^{-1}(y+1) = \lim_{h\to 0} \cos^{-1}(1+h) \Rightarrow$ Not possible

so, Limit does not exists.

Q.4 (3)

$\lim_{x\to 0}\left\{\cot\left(\frac{\pi}{4}+x\right)\right\}^{(\operatorname{cosec} x)}$

$\ell = e^{\lim_{x\to 0} \operatorname{cosec} x\left(\cot\left(\frac{\pi}{4}+x\right)-1\right)}$

$= e^{\lim_{x\to 0} \operatorname{cosec} x\left(\frac{1}{\tan(\frac{\pi}{4}+x)}-1\right)}$

$= e^{\lim_{x\to 0} \operatorname{cosec} x\left(\frac{1-\tan x-1-\tan x}{1+\tan x}\right)}$

$= e^{\lim_{x\to 0} -\operatorname{cosec} x\left(\frac{2\tan x}{1+\tan x}\right)} = e^{-\lim_{x\to 0}\frac{2}{\cos x+\sin x}} = e^{-2}$

Q.5 **(2)**

$\ell im_{x\to 1}\frac{\left(\sum_{k=1}^{100} x^k\right)-100}{x-1}$

$= \ell im_{x\to 1}\left[\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+.....+\frac{x^{100}-1}{x-1}\right]$

$= 1 + 2 + 3 + ... + 100 = \frac{(100).(101)}{2} = 5050$

Q.6 **(3)**

$$\ell im_{x\to\frac{\pi}{2}} \tan^2 x \left(\sqrt{2\sin^2x+3\sin x+4}-\sqrt{\sin^2x+6\sin x+2}\right)$$

$$= \ell im_{x\to\frac{\pi}{2}} \tan^2 x \left(\frac{2\sin^2x+3\sin x+4-(\sin^2x+6\sin x+2)}{\sqrt{2\sin^2x+3\sin x+4}+\sqrt{\sin^2x+6\sin x+2}}\right)$$

$$= \ell im_{x\to\frac{\pi}{2}} \tan^2 x \,.\, \frac{(\sin^2 x-3\sin x+2)}{\sqrt{2+3+4}+\sqrt{1+6+2}}$$

(put $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ & in direction sin x can be taken as 1)

$$= \ell im_{x\to\frac{\pi}{2}} \frac{1}{6}\left[\frac{\sin^2 x-3\sin x+2}{\cos^2 x}\right]\left(\frac{0}{0}\text{ form}\right)$$

(use L'Hospital rule)

$$= \frac{1}{6}\ell im_{x\to\frac{\pi}{2}} \frac{2\sin x\cos x-3\cos x}{2\cos x(-\sin x)}$$

$$= \frac{1}{6}\ell im_{x\to\frac{\pi}{2}} \frac{2\sin x-3}{-2\sin x}$$

$$= \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$$

Q.7 (4)

$$\lim_{n\to\infty} [(1+x)(1+x^2)\ldots\ldots.(1+x^{2^n})]$$

Multipling (1 – x) in N^r & D^r

$$= \lim_{n\to\infty}\left[\frac{(1-x)(1+x)\ldots\ldots..(1+x^{2^n})}{(1-x)}\right] = \lim_{n\to\infty}\left(\frac{1-x^{2^{n+1}}}{1-x}\right)$$

$$= \frac{1}{(1-x)}\lim_{n\to\infty}\left(1-x^{2^{n+1}}\right) = \frac{1}{(1-x)}$$

$\because$ $|x \le 1|$

Q.8 (3)

$$\ell im_{x\to3} \frac{(x^3+27)\,\ell n(x-2)}{(x^2-9)}$$

$$= \ell im_{x\to3} \frac{(x+3)(x^2-3x+9)\,\ell n[1+(x-3)]}{(x-3)(x+3)} = \ell im_{x\to3} (x^2-3x+9).\frac{\ell n[1+(x-3)]}{(x-3)}$$

= (9 – 9 + 9) (1) = 9

Q.9 **(2)**

$$\ell im_{x\to0} \frac{(4^x-1)^3}{\sin\left(\frac{x}{p}\right)\ell n\left(1+\frac{x^2}{3}\right)}$$

$$= \ell im_{x\to0} \frac{x^3\left(\frac{4^x-1}{x}\right)^3}{\left(\frac{x}{p}\right)\left[\frac{\sin\left(\frac{x}{p}\right)}{\frac{x}{p}}\right].\ell n\left(1+\frac{x^2}{3}\right)}$$

$$= 3p\,.\,\ell im_{x\to0} \frac{\left(\frac{4^x-1}{x}\right)^3}{\left[\frac{\sin\left(\frac{x}{p}\right)}{\frac{x}{p}}\right]}\cdot\frac{\left(\frac{x^2}{3}\right)}{\ell n\left(1+\frac{x^2}{3}\right)}$$

= 3 p (ℓn 4)^3

Q.10 **(4)**

$$\text{Lim}_{x\to2} \frac{\sin\left(e^{x-2}-1\right)}{\ell n\,(x-1)}$$

$$= \text{Lim}_{x\to2} \frac{\sin(e^{x-2}-1)}{(e^{x-2}-1)}\times\frac{(e^{x-2}-1)}{\ell n\,(x-1)}$$

$$= \text{Lim}_{x\to2} \frac{e^{(x-2)}-1}{(x-2)}\times\frac{(x-2)}{\ell n\,(1+(x-2))} = 1\times1 = 1$$

Aliter : → we can do the same question by L – Hospital rule also,

$$\text{Lim}_{x\to2} \frac{\sin(e^{(x-2)}-1)}{\ell n\,(x-1)} = \text{Lim}_{x\to2} \frac{\cos(e^{(x-2)}-1)\times e^{(x-2)}}{\frac{1}{(x-1)}}$$

$$= \text{Lim}_{x\to2} (x-1)\,.\,e^{(x-2)}\,.\,\cos(e^{x-2}-1) = 1$$

Q.11 **(2)**

$$\lim_{x\to-\infty} \frac{x^2\sin\left(\frac{1}{x}\right)}{\sqrt{9x^2+x+1}}$$

Put $x = -\frac{1}{y}$, so limit changes to $y \to 0$

$$\lim_{y\to 0} \frac{-\sin y}{y^2\sqrt{\frac{9}{y^2}-\frac{1}{y}+1}} = \lim_{y\to 0} \frac{-\sin y}{y\sqrt{y^2-y+9}}$$

$$= -\frac{1}{\sqrt{9}} = -\frac{1}{3}$$

Q.12 (4)

$$\lim_{x\to 0} \frac{\sin(\ell n(1+x))}{\ell n(1+\sin x)}$$

$$= \lim_{x\to 0} \frac{\sin(\ell n(1+x))}{\ell n(1+x)} \times \frac{\ell n(1+x)}{x} \times \frac{\sin x}{\ell n(1+\sin x)}$$

$= 1 . 1 . 1 = 1$

Q.13 (3)

$$\lim_{x\to\frac{\pi}{2}} \left[\frac{x-\frac{\pi}{2}}{\cos x}\right] \text{ Let } x = \left(\frac{\pi}{2}+h\right)$$

$$= \lim_{h\to 0} \left[\frac{\frac{\pi}{2}+h-\frac{\pi}{2}}{\cos\left(\frac{\pi}{2}+h\right)}\right] = \lim_{h\to 0}\left[\frac{h}{-\sinh}\right] = -2$$

Q.14 **(3)**

$$\lim_{x\to\infty} \frac{x^3\sin\left(\frac{1}{x}\right)+x+1}{x^2+x+1}$$

$$\lim_{x\to\infty} \frac{x^2\left\{\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}+\frac{1}{x}+\frac{1}{x^2}\right\}}{x^2\left\{1+\frac{1}{x}+\frac{1}{x^2}\right\}} = \frac{1}{1} = 1$$

Q.15 **(1)**

$$\lim_{n\to\infty} \frac{-3n+(-1)^n}{4n-(-1)^n} = \lim_{n\to\infty} \frac{-3+\frac{(-1)^n}{n}}{4-\frac{(-1)^n}{n}}$$

$$= \lim_{n\to\infty} \frac{-3+\frac{(-1 \text{ or } 1)}{\infty}}{4-\frac{(-1 \text{ or } 1)}{\infty}} = -\frac{3}{4}$$

Q.16 **(3)**

$$\lim_{\theta\to 0}\left(\left[\frac{n\sin\theta}{\theta}\right]+\left[\frac{n\tan\theta}{\theta}\right]\right)$$

As we know,

$$\frac{\sin\theta}{\theta} < 1 \Rightarrow \frac{n\sin\theta}{\theta} < n \Rightarrow \left[\frac{n\sin\theta}{\theta}\right] = (n-1)\text{...(1)}$$

Also, $\frac{\tan\theta}{\theta} > 1 \Rightarrow \frac{n\tan\theta}{\theta} > n \Rightarrow \left[\frac{n\tan\theta}{\theta}\right] = n\text{...(2)}$

Now, $\lim_{\theta\to 0}\left(\left[\frac{n\sin\theta}{\theta}\right]+\left[\frac{n\tan\theta}{\theta}\right]\right) = n-1+n = (2n-1)$

Q.17 (2)

$$\ell im_{n\to\infty}\ n\cos\left(\frac{\pi}{4n}\right)\ \sin\left(\frac{\pi}{4n}\right)$$

$$= \ell im_{n\to\infty} \cos\left(\frac{\pi}{4n}\right)\ \ell im_{n\to\infty} \frac{\sin\left(\frac{\pi}{4n}\right)}{\left(\frac{\pi}{4n}\right)} \times \frac{\pi}{4}$$

$$= 1 . 1 . \frac{\pi}{4} = \frac{\pi}{4}$$

Q.18 (4)

$$\ell im_{n\to\infty} \frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}$$

$$= \ell im_{n\to\infty} \frac{5.5^n+3^n-4^n}{5^n+2^n+27.9^n}$$

$$= \ell im_{n\to\infty} \frac{5.\left(\frac{5}{9}\right)^n+\left(\frac{3}{9}\right)^n-\left(\frac{4}{9}\right)^n}{\left(\frac{5}{9}\right)^n+\left(\frac{2}{9}\right)^n+27}$$

$$= \frac{0+0-0}{0+0+27} = 0$$

Q.19 **(3)**

$$\lim_{x\to 0}\left[\frac{\sin[x-3]}{[x-3]}\right]$$

LHL at x = 0

$$\lim_{h\to 0}\left[\frac{\sin[-3-h]}{[-3-h]}\right] = \lim_{h\to 0}\left[\frac{\sin(-4)}{(-4)}\right] = -1$$

RHL at x = 0

$$\lim_{h\to 0}\left[\frac{\sin[3+h]}{[3+h]}\right] = \lim_{h\to 0}\left[\frac{\sin 3}{3}\right] = 1$$

since LHL ≠ RHL hence limit does not exist.

Q.20 (1)

$$\lim_{x\to\infty}\left(\frac{x+2}{x-2}\right)^{(x+1)}$$ It is of the form 1^∞, so

$$\ell = e^{\lim\limits_{x\to\infty}(x+1)\left[\frac{x+2}{x-2}-1\right]} = e^{\lim\limits_{x\to\infty}\left(\frac{x+1}{x-2}\right)[x+2-x+2]}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{1+1/x}{1-2/x}\right).4} = e^4$$

Q.21 (1)

$$\lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{5}{x}}$$ It is of the form 1^∞, so

$$\ell = e^{\lim\limits_{x\to 0^+}\frac{5}{x}\left(1+\tan^2\sqrt{x}-1\right)} = e^{\lim\limits_{x\to 0^+}\frac{5}{x}\left(\tan^2\sqrt{x}\right)}$$

$$= e^{\lim\limits_{h\to 0}\frac{5}{h}\left(\tan^2\sqrt{h}\right)} = e^{5\lim\limits_{h\to 0}\left(\frac{\tan^2\sqrt{h}}{\left(\sqrt{h}\right)^2}\right)} = e^5$$

Q.22 **(2)**

$$\lim_{x\to\frac{\pi}{4}}(1+[x])^{1/\ell n\tan x}$$

After putting limit [x] becomes zero, so base is dot one hence $1^\infty = 1$

Q.23 **(2)**

$$\lim_{x\to 0^+}\frac{\cos^{-1}(1-x)}{\sqrt{x}}$$

$$= \lim_{h\to 0}\frac{\cos^{-1}[1-(0+h)]}{\sqrt{0+h}}$$

$$= \lim_{h\to 0}\frac{\cos^{-1}(1-h)}{\sqrt{h}}$$

Let 1 – h = cos θ

$\sin\theta = \sqrt{1-(1-h)^2}$

$\therefore\ \theta = \sin^{-1}\sqrt{2h-h^2}$

$$= \lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{h}}$$

$$= \lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{2h-h^2}}\cdot\frac{\sqrt{2h-h^2}}{\sqrt{h}}$$

$= 1 \times \sqrt{2} = \sqrt{2}$

Q.24 **(2)**

$$\lim_{x\to 0}\frac{\sin(6x^2)}{\ell n\cos(2x^2-x)}$$

$$= \lim_{x\to 0}\frac{\sin 6x^2}{6x^2}\cdot\frac{6x^2}{\ell n(\cos(2x^2-x))}$$

$= 1.\ \lim\limits_{x\to 0}\dfrac{6x^2}{\ell n(\cos(2x^2-x))}\left(\dfrac{0}{0}\text{ form}\right)$ [Using L' Hospital rule]

$$= \lim_{x\to 0}\frac{12x}{\dfrac{(-\sin(2x^2-x))(4x-1)}{\cos(2x^2-x)}}$$

$$= \lim_{x\to 0}\frac{-12\cos(2x^2-x)}{4x-1}\cdot\frac{x(2x-1)}{\sin(2x^2-x)}\frac{1}{(2x+1)}$$

$$= \frac{1-12}{1.1} = -12$$

Q.25 **(1)**

$$\lim_{x\to\infty}\left(x-x^2\,\ell n\left(1+\frac{1}{x}\right)\right)$$

$$= \lim_{x\to\infty} x - x^2\left(\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{3x^3}-......\right)$$

$$= \lim_{x\to\infty} x - x + \frac{1}{2} - \frac{1}{3x} + ...= \frac{1}{2}$$

Q.26 **(3)**

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$ It is of the form 1^∞, so,

$$\Rightarrow \ell = e^{\lim\limits_{x\to\infty}x\left[\frac{x^2-2x+1}{x^2-4x+2}-1\right]} = e^{\lim\limits_{x\to\infty}\left(\frac{2x^2-x}{x^2-4x+2}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{2-1/x}{1-4/x+2/x^2}\right)} = e^2$$

Q.27 **(3)**

$$\lim_{x\to a}\left(2-\frac{a}{x}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$$ It is of the form 1^∞,

$\ell = e^{\lim\limits_{x\to a} \tan\frac{\pi x}{2a}\left(2-\frac{a}{x}-1\right)} = e^{\lim\limits_{x\to a} \tan\frac{\pi x}{2a}\left(\frac{x-a}{x}\right)}$

Put x = a + h,

$\ell = e^{\lim\limits_{h\to 0} \tan\left(\frac{\pi}{2}+\frac{\pi h}{2a}\right).\left(\frac{h}{a+h}\right)} = e^{-\lim\limits_{h\to 0} \cot\left(\frac{\pi h}{2a}\right).\left(\frac{h}{a+h}\right)}$

$= e^{-\lim\limits_{h\to 0} \frac{1}{\tan\left(\frac{\pi h}{2a}\right)}\times(\pi h/2a)\times\frac{2a/\pi}{(a+h)}} = e^{-\lim\limits_{h\to 0} \frac{2a}{\pi}\times\frac{1}{a+h}} = e^{-2/\pi}$

Q.28 (1)

$\lim\limits_{x\to 0}(\cos x)^{\frac{1}{\sin x}}$ (1^∞ form)

$= e^{\lim\limits_{x\to 0}\left(\frac{\cos x-1}{\sin x}\right)}$

$= e^{\lim\limits_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}}$

$= e^{\lim\limits_{x\to 0}-\tan\left(\frac{x}{2}\right)}$

$= e^0 = 1$

Q.29 (3)

$\lim\limits_{x\to\frac{\pi}{2}} \frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$ Put $x = \left(\frac{\pi}{2}+h\right)$

$= \lim\limits_{h\to 0} \frac{1-\tan\left(\frac{\pi}{4}+\frac{h}{2}\right)}{1+\tan\left(\frac{\pi}{4}+\frac{h}{2}\right)} \times \frac{1-\sin\left(\frac{\pi}{2}+h\right)}{\left(\pi-\frac{2\pi}{2}-2h\right)^3}$

$= \lim\limits_{h\to 0} \frac{1-\left\{\frac{1+\tan\frac{h}{2}}{1-\tan\frac{h}{2}}\right\}}{1+\left\{\frac{1+\tan\frac{h}{2}}{1-\tan\frac{h}{2}}\right\}} \left(\frac{1-\cos h}{-8h^3}\right)$

$= \lim\limits_{h\to 0} \left(\frac{-2\tan(h/2)}{-2\times 8h^3}\right) \times (2\sin^2(h/2))$

$= \frac{1}{4}\lim\limits_{h\to 0}\left(\frac{\tan h/2}{(h/2).2}\right)\times\left(\frac{\sin^2(h/2)}{h^2/4\times 4}\right) = \frac{1}{32}$

Q.30 (3)

$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\Rightarrow \lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} x\sin\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right)$

Put $x = \frac{1}{y} \Rightarrow \lim\limits_{y\to 0}\left(\frac{\sin y}{y}\right) + \lim\limits_{y\to 0}\sin y^2 = 1+0 = 1$

Q.31 (1)

$\text{Lim}_{x\to\frac{\pi}{2}} \frac{1-2^{\cos x}}{x\left(x-\frac{\pi}{2}\right)\, 2^{\cos x}}$

$= \text{Lim}_{x\to\frac{\pi}{2}} \frac{2^{\sin\left(\frac{\pi}{2}-x\right)}-1}{\left(\frac{\pi}{2}-x\right)x.2^{\cos x}} = \frac{2\ln 2}{\pi}$

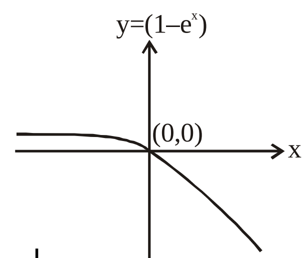
Q.32 (2)

$\lim\limits_{x\to 0}(\cos mx)^{\frac{n}{x^2}}$ since it is of the form 1^∞, so,

$\ell = e^{\lim\limits_{x\to 0}\frac{n}{x^2}(\cos mx-1)} = e^{\lim\limits_{x\to 0}\frac{-2n\sin^2\left(\frac{mx}{2}\right)}{x^2\times\frac{m^2}{4}\times\frac{4}{m^2}}}$

$= e^{\left(-\frac{m^2 n}{2}\right)}$

Q.33 **(A)**



$\lim\limits_{x\to 0}\left[(1-e^x)\frac{\sin x}{|x|}\right]$

RHL

LHL

$\lim\limits_{x\to 0^+}\left[(1-e^x)\frac{\sin x}{x}\right]$

$= (-0.99) = -1$

$\lim\limits_{x\to 0^-}\left[(1-e^x)\frac{\sin x}{-x}\right]$

$= (-0.9999) = -1$

Q.34 (3)

$$f(x) = \begin{cases} \dfrac{\tan^2[x]}{(x^2-[x]^2)} & ; x > 0 \\ 1 & ; x = 0 \\ \sqrt{\{x\}\cot\{x\}} & ; x < 0 \end{cases}$$

RHL : $\lim\limits_{x\to 0^-} \sqrt{\{x\}\cot\{x\}} = \sqrt{1\times\cot 1} = \sqrt{\cot 1}$

LHL : $\lim\limits_{x\to 0^+} \dfrac{\tan^2[x]}{(x^2-[x]^2)} = 0 \Rightarrow \cot^{-1}$

$$\left(\lim_{x\to 0^-} f(x)\right)^2 = 1$$

Q.35 (3)

$$\ell = \lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$$

$$\ell = \lim_{x\to 0} \frac{(1-\cos x\sqrt{\cos 2x})(1+\cos x\sqrt{\cos 2x})}{x^2(1+\cos x\sqrt{\cos 2x})}$$

$$\ell = \lim_{x\to 0} \frac{1-\cos^2 x.\cos 2x}{x^2(1+1\sqrt{1})}$$

$$\ell = \lim_{x\to 0} \frac{1-\cos^2 x(1-2\sin^2 x)}{2x^2}$$

$$\ell = \lim_{x\to 0} \frac{1-\cos^2 x+2\sin^2 x\cos^2 x}{2x^2}$$

$$\ell = \lim_{x\to 0} \frac{\sin^2 x}{2x^2} + \lim_{x\to 0} \frac{\sin^2 x}{x^2}.\cos^2 x$$

$$\ell = \frac{1}{2} + 1 = \frac{3}{2}$$

Q.36 (2)

$$\lim_{x\to 0} \frac{\cos(\sin x)-\cos x}{x^4}$$

$$= \lim_{x\to 0} \frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$= \lim_{x\to 0} \frac{2\sin\left\{\frac{\sin x+x}{2}\right\}}{\left\{\frac{\sin x+x}{2}\right\}} \times \frac{\left(\frac{\sin x+x}{2}\right)}{x^4} \times \frac{\sin\left(\frac{x-\sin x}{2}\right)}{\dfrac{\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}}$$

$$= \lim_{x\to 0} 2\left(\frac{x+\sin x}{2}\right) \times \left(\frac{x-\sin x}{2}\right) \times \frac{1}{x^4}$$

$$= \lim_{x\to 0} \frac{1}{2}\left(\frac{x+\sin x}{x}\right)\left(\frac{x-\sin x}{x^3}\right) =$$

$$\lim_{x\to 0} \frac{1}{2}\left(1+\frac{\sin x}{x}\right)\cdot\left(\frac{x-\sin x}{x^3}\right)$$

$$= \frac{1}{2}(1+1)\cdot\left(\frac{1}{6}\right) = \frac{1}{6}$$

Q.37 (2)

$\lim\limits_{x\to\infty} \left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x} = e^2$ It is of the form 1^∞

$$\ell = e^{\lim\limits_{x\to\infty} 2x\left(1+\frac{a}{x}+\frac{b}{x^2}-1\right)} = e^{\lim\limits_{x\to\infty}\left(2a+\frac{2b}{x}\right)} = e^{2a}$$

but $\ell = e^2$, so $e^{2a} = e^2 \Rightarrow a = 1$ & $b \in R$

Q.38 (1)

$\ell = \lim\limits_{x\to\infty} \left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^x$ It is of form 1^∞

$$\ell = e^{\lim\limits_{x\to\infty} x\left\{\frac{x^2+5x+3}{x^2+x+3}-1\right\}} = e^{\lim\limits_{x\to\infty} x\left(\frac{4x}{x^2+x+3}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{4}{1+1/x+3/x^2}\right)} = e^4$$

Q.39 (3)

$$\lim_{x\to 0} \frac{2\left\{\sqrt{3}\sin\left(\frac{\pi}{6}+x\right)-\cos\left(\frac{\pi}{6}+x\right)\right\}}{x\sqrt{3}(\sqrt{3}\cos x-\sin x)}$$

$$= \lim_{x\to 0} \frac{2\times 2\left\{\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+x\right)-\frac{1}{2}\cos\left(\frac{\pi}{6}+x\right)\right\}}{x\sqrt{3}(\sqrt{3}\cos x-\sin x)}$$

$$= \lim_{x\to 0} \frac{4\times \sin\left(\frac{\pi}{6}+x-\frac{\pi}{6}\right)}{x\sqrt{3}\,(\sqrt{3}\cos x-\sin x)}$$

$$= \lim_{x\to 0} 2\frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)}$$

$$= 2\lim_{x\to 0} \frac{1}{\sqrt{3}\cos\left(\frac{\pi}{6}+x\right)} = \frac{4}{3}$$

Q.40 (3)

$$\ell = \lim_{x\to 0} \frac{\sin\cdot[\cos x]}{1+[\cos x]}$$

LHL : $\lim_{h\to 0} \frac{\sin[\cos(0-h)]}{1+[\cos(0-h)]} \Rightarrow \lim_{h\to 0} \frac{\sin 0}{1+0} = 0$

($\because$ [cos (0 – h)] = 0)

RHL : $\lim_{h\to 0} \frac{\sin[\cos(0+h)]}{1+[\cos(0+h)]} \Rightarrow \lim_{h\to 0} \frac{\sin 0}{1+0} = 0$

so, $\lim_{x\to 0} \frac{\sin[\cos x]}{1+[\cos x]} = 0$

Q.41 (1)

1^{∞} From

$$\ell = \lim_{x\to\infty} x\left[\sin\frac{1}{x}+\cos\frac{1}{x}-1\right]$$

put x = 1/t

$$= \lim_{t\to 0} \frac{1}{t}[\sin t + \cos t - 1]$$

$$= \lim_{t\to 0} \frac{1}{t}\left[2\sin\frac{t}{2}\cos\frac{t}{2}-2\sin^2\frac{t}{2}\right]$$

$$= \lim_{t\to 0} \frac{1}{t}\ 2\sin\frac{t}{2}\left[\cos\frac{t}{2}-\sin\frac{1}{2}\right] = 1 = e^1$$

Q.42 (3)

$$\ell im_{x\to 0} \frac{e^{-\frac{x^2}{2}}-\cos x}{x^3\sin x}$$

$$= \ell im_{x\to 0} \frac{\left[1-\frac{x^2}{2}+\frac{x^4}{4.2!}-......\right]-\left[1-\frac{x^2}{2!}+\frac{x^4}{4!}-......\right]}{x^3\left[x-\frac{x^3}{3!}+.......\right]}$$

$$= \ell im_{x\to 0} \frac{x^4\left[\frac{1}{8}-\frac{1}{4!}\right]-x^6\left[\frac{1}{8.3!}-\frac{1}{6!}\right]+.....}{x^4\left[1-\frac{x^2}{3!}+.......\right]}$$

$$= \left[\frac{1}{8}-\frac{1}{24}\right] = \frac{1}{12}$$

Q.43 (3)

$$\ell = \lim_{x\to 0} \frac{e^{-nx}+e^{nx}-2\cos\frac{nx}{2}-kx^2}{(\sin x-\tan x)}$$

Using series expansion

=

$$\lim_{x\to 0} \frac{1+\frac{nx}{1!}+\frac{n^2x^2}{2!}+\frac{n^3x^3}{3!}+1-\frac{nx}{1!}+\frac{n^2x^2}{2!}-\frac{n^3x^3}{3!}-2\cos\frac{nx}{2}-kx^2}{\left\{\frac{(\sin x-\tan x)}{x^3}\right\}x^3}$$

$$= \lim_{x\to 0} \frac{2+n^2x^2+\frac{n^4x^4}{12}-2\left(1-\frac{n^2x^2}{4.2!}+\frac{n^4x^4}{16.4!}\right)-kx^2}{x^3\left(\frac{\sin x-\tan x}{x^3}\right)} =$$

$$\frac{\left(\frac{5n^2}{4}-k\right)x^2+Mx^4}{x^3\left(\frac{\sin x-\tan x}{x^3}\right)}$$

For existence of limit, $\frac{5n^2}{4}$ = k, now check value of n & k from options.

Q.44 (4)

$$\lim_{n\to\infty} \frac{1}{n^4}\left([1^3x]+[2^3x]+.....+[n^3x]\right)$$

$1^3x - 1 < [1^3x] \le 1^3x$

$2^3x - 1 < [2^3x] \le 2^3x$

$3^3x - 1 < [3^3x] \le 3^3x$

⋮ ⋮ ⋮

$n^3x - 1 < [n^3x] \le n^3x$ so,

$(1^3x + ... + n^3x) - n < [1^3x] ... + [n^3x] \le 1^3x + 2^3x + ... n^3x$

$$\lim_{n\to\infty} \frac{(1^3x+...+n^3x)-n}{n^4} < \lim_{n\to\infty}$$

$$\ell \le \lim_{n\to\infty} \frac{1^3x+...+n^3x}{n^4}$$

$$\lim_{n\to\infty}\frac{\left(\frac{n(n+1)}{2}\right)^2 x-n}{n^4}<\lim_{n\to\infty}$$

$$\ell\le\lim_{n\to\infty}\frac{\left(\frac{n(n+1)}{2}\right)^2 x}{n^4}$$

$$\lim_{n\to\infty}\left(\frac{n^4\left(1+\frac{1}{n}\right)^2 x}{4n^4}-\frac{1}{n^3}\right)<\lim_{n\to\infty}$$

$$\ell\le\lim_{n\to\infty}\frac{n^4\left(1+\frac{1}{n}\right)^2 x}{4n^4}$$

$$\frac{x}{4}-0<\lim_{n\to\infty}\ell<\frac{x}{4}\quad \text{So, } \lim_{n\to\infty}\ell=\frac{x}{4}$$

Q.45 (4)

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+......+\frac{1}{\sqrt{n^2+2n}}\right)$$

$$\Rightarrow \lim_{n\to\infty}\frac{(2n+1)}{\sqrt{n^2+2n}}\le\ell\le\frac{(2n+1)}{\sqrt{n^2}}$$

$$\Rightarrow \lim_{n\to\infty}\frac{(2n+1)}{n\sqrt{1+\frac{2n}{n^2}}}\le\ell\le\lim_{n\to\infty}\left(\frac{2n+1}{n}\right)$$

$$\Rightarrow\ 2\le\ell\le2\Rightarrow\ell=2$$

Q.46 (2)

Given : $f(x)=\begin{cases}x-1, & x\ge1\\ 2x^2-2, & x<1\end{cases}$

$g(x)=\begin{cases}x+1, & x>0\\ -x^2+1, & x\le0\end{cases}$ & $h(x)=|x|$

LHL : $\lim_{x\to0^-} f(g(h(0^-)))=\lim_{x\to0^-} f(g(0^+))$

$=\lim_{x\to0^-} f(1^+)=0$

RHL : $\lim_{x\to0^+} f(g(h(x)))=\lim_{x\to0^+} f(g(h(0^+)))\ =\lim_{x\to0^+}$ $f\{g(0^+)\}=0$

Q.47 **(3)**

$$f(x)=\lim_{t\to0}\left(\frac{2x}{\pi}\cot^{-1}\left(\frac{x}{t^2}\right)\right)$$

<u>Case –1</u> : → If $x<0$,
Let $x=-k$, where $k>0$.

$$\text{so, } \frac{2x}{\pi}\cot^{-1}\left(\frac{x}{t^2}\right)=\frac{-2k}{\pi}\cot^{-1}\left(\frac{-k}{t^2}\right)=\frac{-2k}{\pi}\left(\frac{\pi}{2}+\tan^{-1}\frac{k}{t^2}\right)$$

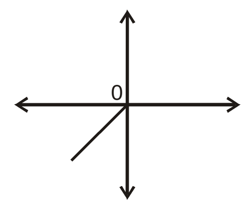
$$\lim_{t\to0}\frac{2x}{\pi}\cot^{-1}\left(\frac{x}{t^2}\right)=\lim_{t\to0}-\frac{2k}{\pi}\left(\frac{\pi}{2}+\tan^{-1}\frac{k}{t^2}\right)$$

$= -k-k \quad\Rightarrow$
$2x$

<u>Case – 2</u> : → If $x>0$,

$$\lim_{t\to\infty}\frac{2x}{\pi}\cot^{-1}\left(\frac{x}{t^2}\right)=0$$

i.e. x-axis.
so, graph of f(x) will be



Q.48 **(1)**

Multipling by log x in dinominater & numanater

$$\lim_{x\to\infty}\frac{\log_x n-[x]}{[x]}=\lim_{x\to\infty}\frac{\log n-[x]\log x}{[x]\log x}$$

$$\lim_{x\to\infty}\frac{\log n}{[x]\log x}-1=-1$$

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OBJECTIVE QUESTIONS

Q.1 (B)

$$\ell\text{im}_{x\to\infty}\frac{x^n}{e^x}=0\ (n\in\text{integer})$$

case(i) when n = 0

then $\ell\text{im}_{x\to\infty}\frac{x^n}{e^x}=\ell\text{im}_{x\to\infty}\frac{1}{e^x}=0$

case(ii) when n is +ve integer

$\ell\text{im}_{x\to\infty}\frac{x^n}{e^x}\left(\frac{\infty}{\infty}\text{form}\right)$

$=\ell\text{im}_{x\to\infty}\frac{n!}{e^x}=0$

case(iii) when n is – ve $n=-m$ where $m\in z^+$

$\ell\text{im}_{x\to\infty}\frac{x^n}{e^x}=\ell\text{im}_{x\to\infty}\frac{x^{-m}}{e^x}$

$=\ell\text{im}_{x\to\infty}\frac{1}{x^m.e^x}=0$

so $\displaystyle \lim_{x\to\infty} \frac{x^n}{e^x} = 0$

Q.2 (B)

$$\lim_{x\to a^-}\left(\frac{|x|^3}{a}-\left[\frac{x}{a}\right]^3\right) \quad (a > 0)$$

x = a – h

$$= \lim_{h\to 0}\left(\frac{|a-h|^3}{a}-\left[\frac{a-h}{a}\right]^3\right)$$

$$= \lim_{h\to 0}\left(\frac{|a|^3}{a}-\left[1-\frac{h}{a}\right]^3\right)$$

$= a^2 - 0 = a^2$

Q.3 (A)

Given that , $\displaystyle \lim_{x\to\infty}$ f(x) is finite & non zero.

Also, $\displaystyle \lim_{x\to\infty}\left(f(x)+\frac{3f(x)-1}{f^2(x)}\right)=3$

Let $\displaystyle \lim_{x\to\infty}$ f(x) = ℓ so $\displaystyle \lim_{x\to\infty}\left(f(x)+\frac{3f(x)-1}{f^2(x)}\right) = 3$

$\Rightarrow \ell + \dfrac{3\ell-1}{\ell^2} - 3 = 0 \Rightarrow (\ell-1)^3 = 0 \quad \Rightarrow \ \ell = 1$

Q.4 (D)

$\displaystyle \lim_{x\to a_m}$ $(A_1 . A_2 .... A_n)$, $1 \le m \le n$

$$\lim_{x\to a_m}\frac{x-a_1}{|x-a_1|}.\frac{x-a_2}{|x-a_2|}...\frac{x-a_m}{|x-a_m|}...\frac{x-a_n}{|x-a_n|}$$

$$\lim_{x\to a_m}(1)^{m-1}\times\frac{x-a_m}{|x-a_m|}\times(-1)^{n-m}$$

RHL: $(-1)^{n-m} \times 1$

LHL: $(-1)^{n-m} \times (-1)$

so, limit does not exists.

Q.5 (C)

$$\lim_{n\to\infty}\cos\frac{x}{2}\cos\frac{x}{2^2}\cos\frac{x}{2^3}\cos\frac{x}{2^4}.......\cos\frac{x}{2^n}$$

$$= \lim_{n\to\infty}\frac{\sin x}{2^n\sin\frac{x}{2^n}} = \lim_{n\to\infty}\frac{\sin x}{x}.\left(\frac{\frac{x}{2^n}}{\sin\frac{x}{2^n}}\right) \left(\because \lim_{n\to\infty}\frac{x}{2^n} = 0\right) = \frac{\sin x}{x}$$

Q.6 (C)

α, β are the roots of the equation $ax^2 + bx + c = 0$

$\because \quad ax^2 + bx + c = a(x-\alpha)(x-\beta)$

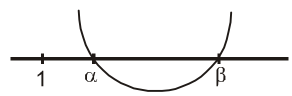
$$\lim_{x\to\alpha}\left(1+ax^2+bx+c\right)^{\frac{1}{x-\alpha}} = e^{\lim_{x\to\alpha}\frac{ax^2+bx+c}{x-\alpha}}$$

$$= e^{\lim_{x\to\alpha}\frac{a(x-\alpha)(x-\beta)}{(x-\alpha)}} = e^{a(\alpha-\beta)}$$

Q.7 (D)

α, β be the roots of the equation $ax^2 + bx + c = 0$ where $1 < \alpha < \beta$.

(i) if a > 0

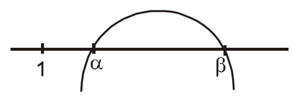


So $ax^2 + bx + c > 0$ when $x \in (-\infty, \alpha) \cup (\beta, \infty)$

So $\displaystyle \lim_{x\to x_0}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$

when a > 0 and $x \in (-\infty, \alpha) \cup (\beta, \infty)$

(ii) If a < 0



So $ax^2 + bx + c > 0$ when $x \in (\alpha, \beta)$

$$\lim_{x\to x_0}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$$

when a < 0 and $x \in (\alpha, \beta)$

So (A) a > 0 and $x_0 < 1$ right

(B) a > 0 and $x_0 > \beta$ right

(C) a < 0 and $\alpha < x_0 < \beta$ right

(D) a < 0 and $x_0 < 1$ wrong

Q.8 (B)

$$\lim_{x\to\infty}\frac{e^x\left[\left(2^{x^n}\right)^{\frac{1}{e^x}}-\left(3^{x^n}\right)^{\frac{1}{e^x}}\right]}{x^n}$$

$$= \lim_{x\to\infty}\frac{(2^{x^n/e^x}-1)-(3^{x^n/e^x}-1)}{(x^n/e^x)}$$

Now, $\displaystyle \lim_{x\to\infty}\frac{x^n}{e^x} = 0$

So, $\displaystyle \lim_{x\to\infty}\left\{\frac{2^{(x^n/e^x)}-1}{(x^n/e^x)}\right\} - \lim_{x\to\infty}\left\{\frac{3^{(x^n/e^x)}-1}{(x^n/e^x)}\right\}$

$= \log_e 2 - \log_e 3 = \log_e\left(\frac{2}{3}\right)$

Q.9 (D)

Given that α & β are the roots of $ax^2 + bx + c = 0$

$$\ell = \lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$

$$\ell = \lim_{x\to\alpha} \frac{2\sin^2\left(\frac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

$$\ell = \lim_{x\to\alpha} \frac{2\sin^2\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)}{\frac{(x-\alpha)^2}{4}\times 4\times\frac{(x-\beta)^2}{(x-\beta)^2}\times\frac{a^2}{a^2}}$$

$$\ell = \frac{2}{4}\times a^2 \lim_{x\to\alpha}(x-\beta)^2 = \frac{a^2}{2}(\alpha-\beta)^2$$

Q.10 (B)

(i) $\ell = \lim_{x\to\infty}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)$

$\Rightarrow \ell$

$$= \lim_{x\to\infty} 2\cos\left(\frac{\sqrt{x+1}+\sqrt{x}}{2}\right).\sin\left(\frac{\sqrt{x+1}-\sqrt{x}}{2}\right)$$

$$\Rightarrow \ell = \lim_{x\to\infty} 2\cos\left(\frac{\sqrt{x+1}+\sqrt{x}}{2}\right).\sin\left(\frac{x+1-x}{2\left(\sqrt{x+1}+\sqrt{x}\right)}\right)$$

$$\Rightarrow \ell = \lim_{x\to\infty} 2\cos\left(\frac{\sqrt{x}+\sqrt{x+1}}{2}\right)\sin\left(\frac{1}{2\left(\sqrt{x}+\sqrt{x+1}\right)}\right)$$

$\Rightarrow \ell$ = (oscillating value –1 to 1) × 0 = 0

(ii) $m = \lim_{x\to-\infty}\left[\sin\sqrt{x+1}-\sin\sqrt{x}\right]$

when $x \to -\infty$

then $\sqrt{x}$ undefined

m is undefined

Q.11 (B)

$(\tan\alpha)\,x + (\sin\alpha)y = \alpha$

$[(\alpha \operatorname{cosec}\alpha)x + (\cos\alpha)y = 1] \times \tan\alpha$

$$x = \frac{\alpha-\tan\alpha}{\tan\alpha-\alpha\sec\alpha}$$

$$x = \lim_{\alpha\to 0} \frac{\alpha-\tan\alpha}{\frac{1}{\cos\alpha}(\sin\alpha-\alpha)} \times \frac{1/\alpha^3}{1/\alpha^3}$$

$$x = \lim_{\alpha\to 0} \frac{-1/3}{-1/6} \times \cos\alpha = 2$$

and $$y = \lim_{\alpha\to 0} \frac{\tan^2\alpha-\alpha^2\sec\alpha}{\sin\alpha(\tan\alpha-\alpha\sec\alpha)}$$

$$= \lim_{\alpha\to 0} \frac{\tan^2\alpha-\alpha^2\sec\alpha+\alpha^2-\alpha^2}{\frac{\sin\alpha}{\alpha}\,\frac{(\tan\alpha-\alpha\sec\alpha)}{\alpha^3}.\alpha^4}$$

$$= \lim_{\alpha\to 0} \frac{\left[\frac{\tan\alpha}{\alpha}+1\right]\left[\frac{\tan\alpha-\alpha}{\alpha^3}\right]-\frac{1-\cos\alpha}{\cos\alpha.\alpha^2}}{\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin\alpha-\alpha}{\alpha^3}\right)\frac{1}{\cos\alpha}} = -1$$

Q.12 (C)

Given $a = \min\{x^2 + 2x + 3, x \in R\}$

$$\& \; b = \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$$

Let $f(x) = x^2 + 2x + 3$. Its discriminant is less than zero. Its min. value is $\frac{-D}{4a} = \frac{-(-8)}{4\times 1} = 2$

Then a = 2 & b

$$= \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2} = \lim_{\theta\to 0}\frac{2\sin^2\left(\frac{\theta}{2}\right)}{\frac{\theta^2}{4}.4} = \frac{1}{2}$$

Now $$\sum_{r=0}^{n} a^r b^{n-r} = \sum_{r=0}^{n} 2^r\left(\frac{1}{2}\right)^{n-r}$$

$$= \frac{1}{2^n}\sum_{r=0}^{n} 2^{2r} = \frac{1}{2^n}(2^0 + 2^2 + ...+ 2^{2n})$$

$$= \frac{1}{2^n}\frac{(2^2)^{n+1}-1}{(2^2-1)} = \frac{4^{n+1}-1}{3\cdot 2^n}.$$

Q.13 (B)

$$\lim_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{x^3}$$

$$= \lim_{x\to 0}\frac{\left[x+\frac{1^2}{3!}x^3+....\right]-\left[x-\frac{x^3}{3}+\frac{x^5}{5}-....\right]}{x^3}$$

$$= \lim_{x \to 0} \frac{x^3\left(\frac{1}{3!}+\frac{1}{3}\right)+x^5\left(\;\right)+......}{x^3}$$

$$= \frac{1}{6}+\frac{1}{3} = \frac{3}{6} = \frac{1}{2}$$

Q.14 (A)

$$\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}(bx-\sin x)} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{\sqrt{a+x}}\lim_{x\to 0}\frac{x^3}{bx-\sin x} = 1$$

$$\Rightarrow \frac{1}{\sqrt{a}}\cdot\lim_{x\to 0}\frac{x^3}{bx-\left[x-\frac{x^3}{3!}+\frac{x^5}{5!}-......\right]} = 1$$

$$\Rightarrow \frac{1}{\sqrt{a}}\cdot\lim_{x\to 0}\frac{x^3}{(b-1)x+\frac{x^3}{3!}-\frac{x^5}{5!}+...} = 1$$

If limit exists, then $b - 1 = 0 \Rightarrow b = 1$

so $\frac{1}{\sqrt{a}}\cdot\lim_{x\to 0}\frac{x^3}{x^3\left[\frac{1}{6}-\frac{x^2}{120}+....\right]} = 1$

$$\Rightarrow \frac{1}{\sqrt{a}}\times\frac{1}{\frac{1}{6}} = 1 \Rightarrow \quad a = 36$$

so $a = 36$, $b = 1$

Q.15 (B)

$$\lim_{y\to 0}\left(\lim_{x\to\infty}\frac{\exp\left(x\,\ell n\left(1+\frac{ay}{x}\right)\right)-\exp\left(x\,\ell n\left(1+\frac{by}{x}\right)\right)}{y}\right)$$

$$= \lim_{y\to 0}\left(\lim_{x\to\infty}\frac{\left(1+\frac{ay}{x}\right)^x-\left(1+\frac{by}{x}\right)^x}{y}\right) \text{ by expansion}$$

$$= \lim_{y\to 0}$$

$$\left(\lim_{x\to\infty}\frac{\left(1+ay+\frac{x(x-1)}{2!}.\frac{a^2y^2}{x^2}+.....\right)-\left(1+by+\frac{x(x-1)}{2!}.\frac{b^2y^2}{x^2}+....\right)}{y}\right)$$

$$= \lim_{y\to 0}\left[\frac{y(a-b)+\frac{y^2}{2}(a^2-b^2)+.....}{y}\right] = a-b$$

Q.16 (A)

$$\lim_{x\to 0^+}\frac{\ell n\sin x}{\ell n\sin\frac{x}{2}}$$

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$$\lim_{x\to 0}\frac{\frac{1}{\sin x}\cos x}{\frac{1}{\sin\frac{x}{2}}\cdot\frac{1}{2}\cdot\cos\left(\frac{x}{2}\right)}$$

$$\lim_{x\to 0}\frac{\left(\frac{x}{\sin x}\right)\cos x}{\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)\cdot\cos\frac{x}{2}} = 1$$

Q.17 (A)

$$f(x) = \sum_{\lambda=1}^{n}\left(x-\frac{1}{\lambda}\right)\left(x-\frac{1}{\lambda+1}\right)$$

$$f(0) = \sum_{\lambda=1}^{n}\left(-\frac{1}{\lambda}\right)\left(-\frac{1}{\lambda+1}\right)$$

$$\Rightarrow f(0) = \sum_{\lambda=1}^{n}\left(\frac{1}{(\lambda)(\lambda+1)}\right)$$

$$\Rightarrow f(0) = \sum_{\lambda=1}^{n}\left(\frac{1}{\lambda}-\frac{1}{\lambda+1}\right) \Rightarrow f(0) = 1-\frac{1}{n+1}$$

$$\Rightarrow f(0) = \frac{n}{n+1}$$

Now $\lim_{n\to\infty} f(0) = \lim_{n\to\infty}\frac{n}{n+1} = \lim_{n\to\infty}\frac{1}{1+\frac{1}{n}} =$

$$\frac{1}{1+0} = 1$$

Q.18 (A)

$$\lim_{n\to\infty}\frac{1.n+2(n-1)+......+n.1}{1^2+2^2+.......+n^2}$$

consider the numerator, it can be written as follows

$$\sum_{r=1}^{n} r\,(n-r+1) = (n+1)\sum_{r=1}^{n} r-\sum_{r=1}^{n} r^2$$

$= (n + 1)(1 + 2 + 3 +....+ n) - (1^2 + 2^2 +...+ n^2)$

$$=\frac{(n+1).n(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}=\frac{n(n+1)(n+2)}{6}$$

Now $\lim\limits_{n\to\infty}\dfrac{\frac{n(n+1)(n+2)}{6}}{\frac{n(n+1)(2n+1)}{6}}=\lim\limits_{n\to\infty}\dfrac{(n+2)}{(2n+1)}=\dfrac{1}{2}$

Q.19 (A)

$\lim\limits_{x\to0} g(f(x))$

L.H.L.$=\lim\limits_{x\to0^-} g[f(x)]$

$\lim\limits_{x\to0^-} g(\sin x)=\lim\limits_{h\to0} g(\sin h)=\lim\limits_{h\to0}(\sin^2 h+1)=1$

R.H.L. $=\lim\limits_{x\to0^+} g[f(x)] = \lim\limits_{x\to0^+} g(\sin x) = \lim\limits_{h\to0}$

$g(\sin h)=\lim\limits_{h\to0}(\sin^2 h+1)=1$

L.H.L. = R.H.L. = 1

so $\lim\limits_{x\to0} g[f(x)]=1$

JEE-ADVANCED
MCQ/COMPREHENSION/MCOLUMN ATCHING

Q.1 **(A, B, C)**

$$f(x)=\frac{x^2-9x+20}{x-[x]}$$

$$\lim_{x\to5^-}\frac{x^2-9x+20}{x-[x]}=\frac{25-45+20}{1}=0$$

$$\lim_{x\to5^+}\frac{x^2-9x+20}{x-[x]}=\lim_{h\to0}\frac{(5+h)^2-9(5+h)+20}{5+h-[5+h]}$$

$$=\lim_{h\to0}\frac{25+10h+h^2-45-9h+20}{h}=\lim_{h\to0}\frac{h^2+h}{h}$$

$$=\lim_{h\to0}\frac{h(h+1)}{h}=1$$

$\because \lim\limits_{x\to5^-} f(x)\neq\lim\limits_{x\to5^+} f(x)$

so $\lim\limits_{x\to5} f(x)$ does not exist

Q.2 **(A, B)**

(A) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2}}{3x-6}=\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2}}{-3x-6}$

$$=\lim_{x\to\infty}\frac{\sqrt{1+\frac{2}{x^2}}}{-3-\frac{6}{x}}=-\frac{1}{3}$$

(B) $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2}}{3x-6}$

$$=\lim_{x\to\infty}\frac{\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}}=\frac{1}{3}$$

Q.3 **(A, B, C, D)**

$$\lim_{x\to\infty}\frac{(ax+1)^n}{x^n+A}$$

(A) If $n\in N$

$$\lim_{x\to\infty}\frac{\left(a+\frac{1}{x}\right)^n}{1+\frac{A}{x^n}}=\frac{(a+0)^n}{1+0}=a^n$$

(B) If $n\in Z^-$ & $a = A = 0$

then $\lim\limits_{x\to\infty}\dfrac{(ax+1)^n}{x^n+A}$

$=\lim\limits_{x\to\infty}\dfrac{1}{x^n}=\infty\ n\in Z^-$

(C) If $n = 0$

then $\lim\limits_{x\to\infty}\dfrac{(ax+1)^n}{x^n+A}$

$$=\lim_{x\to\infty}\frac{1}{1+A}$$

$$=\frac{1}{1+A}$$

(D) If $n\in Z^-$, $A = 0$ & $a\neq0$

then $\lim\limits_{x\to\infty}\dfrac{(ax+1)^n}{x^n+A}=\lim\limits_{x\to\infty}\dfrac{(ax+1)^n}{x^n}$

$$=\lim_{x\to\infty}\left(a+\frac{1}{x}\right)^n$$

$= (a + 0)^n = a^n$

Q.4 (B, C)

$$f(x) = \begin{cases} 1+\dfrac{2x}{a}, & 0 \le x < 1 \\ ax, & 1 \le x < 2 \end{cases}$$

L.H.L. = $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{h\to 0} 1 + \dfrac{2(1-h)}{a} = 1 + \dfrac{2}{a}$

R.H.L. = $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{h\to 0} a(1+h) = a$

$\because \lim\limits_{x\to 1} f(x)$ exists

L.H.L.= R.H.L.

$\Rightarrow 1 + \dfrac{2}{a} = a$

$\Rightarrow a^2 - a - 2 = 0$

$\Rightarrow (a-2)(a+1) = 0$

$\Rightarrow a = 2, -1$

Q.5 **(A, B)**

Let $f(x) = \dfrac{\cos 2 - \cos 2x}{x^2 - |x|}$

(A) $\lim\limits_{x\to -1} f(x)$

for $x = -1$ $|x| = -x$

$f(x) = \dfrac{\cos 2 - \cos 2x}{x^2 + x}$

Now $\lim\limits_{x\to -1} \dfrac{\cos 2 - \cos 2x}{x^2 + x}$ ($\dfrac{0}{0}$ form)

$= \lim\limits_{x\to -1} \dfrac{2\sin 2x}{2x+1} = 2\sin 2$

(B) $\lim\limits_{x\to 1} \dfrac{\cos 2 - \cos 2x}{x^2 - x}$ ($\dfrac{0}{0}$ form) $= \lim\limits_{x\to 1} \dfrac{2\sin 2x}{2x-1}$

$= 2\sin 2$

Q.6 **(A, D)**

For $\lim\limits_{x\to 0} \dfrac{1+a\cos x}{x^2}$

for $\left(\dfrac{0}{0}\right)$ form

$1 + a = 0 \Rightarrow a = -1$

for $\lim\limits_{x\to 0} \dfrac{b\sin x}{x^3} = \lim\limits_{x\to 0} \dfrac{b}{x^2}$

so $b = 0$

Now $\ell = \lim\limits_{x\to 0} \dfrac{1+a\cos x}{x^2} - \lim\limits_{x\to 0} \dfrac{b\sin x}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

$= \lim\limits_{x\to 0} \dfrac{2\sin^2\frac{x}{2}}{x^2}$

$= \lim\limits_{x\to 0} \dfrac{2}{4}\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2$

$= \dfrac{1}{2}.(1)^2 = \dfrac{1}{2}$

$\therefore$ $(a, b) = (-1, 0)$ and $\ell = \dfrac{1}{2}$

Q.7 **(A, B, C, D)**

$\lim\limits_{x\to 0} (\cos x + a\sin bx)^{\frac{1}{x}} = e^2$ (1^∞ form)

$\Rightarrow e^{\lim\limits_{x\to 0}\left(\frac{\cos x + a\sin bx - 1}{x}\right)} = e^2$

$\Rightarrow e^{\lim\limits_{x\to 0}\left(\frac{-\sin x + ab\cos bx}{1}\right)} = e^2$

$\Rightarrow \lim\limits_{x\to 0}(-\sin x + ab\cos bx) = 2$

$\Rightarrow$ $ab = 2$ $\therefore$ $a = 1, b = 2$; $a = 2, b = 1$; $a = 3, b = \dfrac{2}{3}$; $a = \dfrac{2}{3}, b = 3$

Q.8 **(A,B, C)**

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} |0-h|^{\sin(0-h)}$

$= \lim\limits_{h\to 0} h^{\sin(-h)}$

$= e^{\lim\limits_{h\to 0}(-\sin h)(\ell n h)}$

$= e^0 = 1$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to \infty} |0+h|^{\sin(0+h)}$

$= \lim\limits_{h\to \infty} h^{\sin h}$

$= e^{\lim\limits_{h\to\infty}(\sin h)(\ell n h)}$

$= e^0 = 1$

$\because \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = 1$

$\therefore \lim\limits_{x\to 0} f(x) = 1$

Q.9 (A, D)

$\lim\limits_{x\to 0} \dfrac{2\sin x\cos x + a\sin x}{x^3} = p$ (finite)

$\Rightarrow \quad \lim\limits_{x\to 0} \frac{\sin x}{x} \cdot \left(\frac{2\cos x + a}{x^2}\right) = p$

$\Rightarrow \quad \lim\limits_{x\to 0} \frac{\sin x}{x} \cdot \lim\limits_{x\to 0} \frac{2\cos x + a}{x^2} = p$

$\Rightarrow \quad \lim\limits_{x\to 0} \frac{2\cos x + a}{x^2} = p$

For $\left(\frac{0}{0}\right)$ form

$2 + a = 0$
$a = -2$

$\Rightarrow \quad p = \lim\limits_{x\to 0} \frac{2(\cos x - 1)}{x^2} \Rightarrow \quad p = \lim\limits_{x\to 0} \frac{-2\left(2\sin^2 \frac{x}{2}\right)}{x^2}$

$\Rightarrow \quad p = -1$

Q.10 (A, B, C,D)

$f(x) = \frac{|x+\pi|}{\sin x}$

(A) $f(-\pi^+) = \lim\limits_{h\to 0} \frac{|-\pi + h + \pi|}{\sin(-\pi + h)}$

$= \lim\limits_{h\to 0} \frac{|h|}{-\sinh} = -1$

(B) $f(-\pi^-) = \lim\limits_{h\to 0} \frac{|-\pi - h + \pi|}{\sin(-\pi - h)}$

$= \lim\limits_{h\to 0} \frac{|h|}{\sin h} = 1$

(C) $f(-\pi^+) \neq f(-\pi^-)$

so $\lim\limits_{x\to -\pi} f(x)$ does not exist

(D) for $\lim\limits_{x\to \pi} f(x)$

LHL $= \lim\limits_{x\to \pi^-} \frac{|x+\pi|}{\sin x}$

$= \lim\limits_{h\to 0} \frac{2\pi - h}{\sinh} = \frac{2\pi}{0} = \infty$

RHL $= \lim\limits_{x\to \pi^+} \frac{|x+\pi|}{\sin x}$

$= \lim\limits_{h\to 0} \frac{2\pi + h}{-\sinh} = -\frac{2\pi}{0} = -\infty$

LHL $\neq$ RHL

so $\lim\limits_{x\to \pi} f(x)$ does not exist.

Comprehension # 01

Q.11 (A)

Q.12 (D)

Q.13 (A)

(11 to 13)

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{\sin x + ae^x + be^{-x} + c\,\ell n(1+x)}{x^3}$

$= \lim\limits_{x\to 0^+}$

$\frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right) + a\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\right) + b\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots\right) + c\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\right)}{x^3}$

$= \lim\limits_{x\to 0^+} \frac{(a+b) + x(1+a-b) + x^2\left(\frac{a}{2!} + \frac{b}{2!} - \frac{c}{2}\right) + x^3\left(-\frac{1}{3!} + \frac{a}{3!} - \frac{b}{3!} + \frac{c}{3}\right) + x^4\left(\ \right) + \ldots}{x^3}$

$\because$ limit is finite $\Rightarrow \left.\begin{matrix} a+b=0 \\ 1+a-b=0 \\ a+b-c=0 \end{matrix}\right\} \Rightarrow a = -\frac{1}{2}$, $b = \frac{1}{2}$ and $c = 0$

Also $\lim\limits_{x\to 0^+} f(x) = -\frac{1}{3!} + \frac{a}{3!} - \frac{b}{3!} + \frac{c}{3} = -\frac{1}{6}(1 - a + b - 2c) = -\frac{1}{3}$

Comprehension # 02

Q.14 (B)

In radians we can say $\sin x = \sin\left(\frac{\pi x}{180}\right)$

$\therefore \quad \sin x = \sin\left(\pi - \frac{\pi x}{180}\right)$ or $\sin\left(2\pi + \frac{\pi x}{180}\right)$

$\therefore \quad x = \left(\pi - \frac{\pi x}{180}\right)$ or $x = \left(2\pi + \frac{\pi x}{180}\right)$

$x = \frac{180\pi}{180 + \pi}$ or $x = \frac{360\pi}{180 - \pi}$

on comparing with $\frac{p\pi}{q+\pi}$ and $\frac{m\pi}{n-\pi}$

we have, m = 360; n = 180; p = 180 and q = 180
$\therefore$ m + n + p + q = 900

Also $\frac{mn}{pq} = \frac{360 \cdot 180}{180 \cdot 180} = 2 = L$

Q.15 (A)

$$\lim_{x\to\infty} \frac{(Ax^2+Bx)-C^2x^2}{\sqrt{Ax^2+Bx}+Cx} = 2;$$

$$\lim_{x\to\infty} \frac{(A-C^2)x^2+Bx}{x\left[\sqrt{A+(B/x)}+C\right]} = 2$$

for existence of limit $A = C^2$

hence $\frac{BC}{A} = \frac{B}{C}$ (using $A = C^2$)

$\therefore$ $L = \frac{B}{\sqrt{A}+C} = 2$

if $C = -\sqrt{A}$ then limit does not exist hence $C = \sqrt{A}$

$\therefore$ $\frac{B}{2C} = 2 \Rightarrow \frac{B}{C} = 4$

Q.16 (C)

g ' (x) = – 2 f ' (10 – 2x)

g ' (L = 2) = – 2 f ' (6) = 0

Q.17 (A) → (s),(B) → (r), (C) → (p), (D) → (q),

(A) If m > n ⇒ m – n > 0 ∴ $\lim_{x\to 0}\phi(x) = 0$

(B) If m = n, then

$$\lim_{x\to 0}\phi(x) = \lim_{x\to 0}$$

$$\frac{a_0x^m + a_1x^{m+1} + ... + a_kx^{m+k}}{b_0x^m + b_1x^{m+1} + ... + b_\ell x^{m+\ell}} = \frac{a_0}{b_0}$$

(C) If n – m is even positive, then

$\lim_{x\to 0}\phi(x) = \infty$ as $\frac{a_0}{b_0} > 0$

(D) If n – m is even positive and $\frac{a_0}{b_0} < 0$, then

$\lim_{x\to 0}\phi(x) = -\infty$

Q.18 (A) → (q), (B) → (r), (C) → (s), (D) → (p),

(A) $\lim_{x\to 0} \frac{\tan[e^2]x^2 - \tan[-e^2]x^2}{\sin^2 x}$

$$= \lim_{x\to 0} \frac{[e^2]x^2\frac{\tan[e^2]x^2}{[e^2]x^2} - [-e^2]x^2\frac{\tan[-e^2]x^2}{[-e^2]x^2}}{x^2\frac{\sin^2 x}{x^2}}$$

= [e²] – [– e²]

= 15

(B) $\lim_{x\to 0}\left[\left(\min(t^2+4t+6)\right)\frac{\sin x}{x}\right] = \lim_{x\to 0}\left[\frac{2\sin x}{x}\right]$

∵ sin x < x

$\Rightarrow \frac{2\sin x}{x} < 2 \Rightarrow \left[\frac{2\sin x}{x}\right] = 1$

So $\lim_{x\to 0}\left[\frac{2\sin x}{x}\right] = 1$

(C) $\lim_{x\to 0} \frac{(1+x^2)^{1/3} - (1-2x)^{\frac{1}{4}}}{x+x^2}$

$$= \lim_{x\to 0} \frac{\left(1+\frac{x^2}{3}+....\right) - \left(1-\frac{x}{2}+\frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2!}4x^2+....\right)}{x+x^2} = \frac{1}{2}$$

(D) $\lim_{x\to 0} = \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x} = \lim_{x\to 0}\frac{2\sqrt{2}\sin^2\frac{x}{4}}{\sin^2 x} =$

$$\lim_{x\to 0}\frac{2\sqrt{2}\sin^2\frac{x}{4}}{16.\frac{x^2}{16}.\frac{\sin^2 x}{x^2}} = \frac{\sqrt{2}}{8}$$

NUMERICAL VALUE BASED

Q.1 [2]

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\sin^{-1}(1-\{x\}.\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,(1-\{x\})}$$

$$= \lim_{h\to 0}\frac{\sin^{-1}(1-h).\cos^{-1}(1-h)}{\sqrt{2h}\,(1-h)}$$

$$= \lim_{h\to 0}\frac{\sin^{-1}(1-h)\sin^{-1}\sqrt{h(2-h)}}{(1-h)\,\sqrt{2h}}$$

$$= \lim_{h\to 0}\frac{\frac{\pi}{2}}{1}\frac{\sin^{-1}\sqrt{h(2-h)}}{\sqrt{2h-h^2}\,\sqrt{2h}}\sqrt{2h-h^2}$$

$$= \lim_{h\to 0} \frac{\pi}{2} \cdot 1 \cdot \sqrt{1-\frac{h}{2}} = \frac{\pi}{2}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin^{-1}(1-\{x\})\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,(1-\{x\})}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}h \cdot \cos^{-1}h}{\sqrt{2(1-h)}\,h} = \frac{\frac{\pi}{2}}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

Q.2 [1]

$$\lim_{x\to\infty} \left(x\sin\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right)\right)$$

$$\lim_{x\to\infty} \left(\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} + \sin\left(\frac{1}{x^2}\right)\right) = 1 + 0 = 1$$

Q.3 [2]

$$\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x\cos 2x}{(1+\cos x\sqrt{\cos 2x})x^2}$$

$$= \lim_{x\to 0} \frac{1}{(1+\cos x\sqrt{\cos 2x})} \cdot \lim_{x\to 0} \frac{1-\cos^2 x\cos 2x}{x^2}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{1-\left(\frac{1+\cos 2x}{2}\right)\cos 2x}{x^2}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{2-\cos 2x-(\cos 2x)^2}{2x^2}$$

$$= \frac{1}{4}\lim_{x\to 0} -\frac{1}{x^2}(\cos 2x + 2)(\cos 2x - 1)$$

$$= \frac{1}{4}\lim_{x\to 0}(\cos 2x + 2). \lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

$$= \frac{(1+2)}{4} \cdot \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = \frac{3}{4}.2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = \frac{3}{2}$$

$\Rightarrow$ limit $= \frac{3}{2} + \frac{1}{2} = 2$

Q.4 [1]

$\lim_{x\to\infty} f(x)$ exist and is finite & non zero

$$\lim_{x\to\infty} \left[f(x) + \frac{3f(x)-1}{f^2(x)}\right] = 3 \quad \Rightarrow \quad \lim_{x\to\infty}$$

$$f(x) + \frac{3\lim_{x\to\infty} f(x) - 1}{\left[\lim_{x\to\infty} f(x)\right]^2} = 3$$

Let $\lim_{x\to\infty} f(x) = A$

$$A + \frac{3A-1}{A^2} = 3$$

$\Rightarrow$ $A = 1$

so $\lim_{x\to\infty} f(x) = 1$

Q.5 [0]

$\lim_{x\to 0} f(g(h(x)))$

L.H.L. $x \to 0^-$

$\lim_{x\to 0^-} h(x) = 0^+$

$\lim_{x\to 0^+} f(g(x))$

then $\lim_{x\to 0^+} g(x) = 1^+$

$\lim_{x\to 1^+} f(x) = 1 - 1 = 0$

R.H.L. $x \to 0^+$

$\lim_{x\to 0^+} h(x) = 0^+$

so $\lim_{x\to 0^+} f(g(x)) = 0$

L.H.L. = R.H.L. = 0

Q.6 [1]

$\lim_{x\to 0} g(f(x))$

L.H.L. $= \lim_{x\to 0^-} g[f(x)]$

$\lim_{x\to 0^-} g(\sin x) = \lim_{h\to 0} g(\sinh) = \lim_{h\to 0} (\sin^2 h + 1) = 1$

R.H.L. $= \lim_{x\to 0^+} g[f(x)] = \lim_{x\to 0^+} g(\sin x) = \lim_{h\to 0}$ $g(\sinh) = \lim_{h\to 0} (\sin^2 h + 1) = 1$

L.H.L. = R.H.L. = 1

so $\lim_{x\to 0} g[f(x)] = 1$

Q.7 [2]

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{n^2+2n}}\right)$$

using sandwitch theorem

$$\frac{1}{\sqrt{n^2}} \le \frac{1}{n}$$

$$\frac{1}{\sqrt{n^2+1}} \le \frac{1}{n}$$

$\vdots \qquad \vdots$

$$\frac{1}{\sqrt{n^2+2n}} \le \frac{1}{n}$$

adding all these inequilities

$$\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{n^2+2n}} \le \frac{2n}{n}$$

Taking both side $\lim\limits_{n\to\infty}$

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{n^2+2n}}\right) = 2$$

Q.8 [1]

$$\lim_{x\to 0^+} x^2\left[\frac{1}{x^2}\right] = \lim_{x\to 0^+} x^2\left(\frac{1}{x^2} - \left\{\frac{1}{x^2}\right\}\right) \Rightarrow$$

$$\lim_{x\to 0^+}\left(1 - x^2\left\{\frac{1}{x^2}\right\}\right) = 1$$

similarly $\lim\limits_{x\to 0^-} f(x) = 1$

Q.9 [11]

$$\lim_{x\to 0}\left(\frac{\sin^{-1}x - \tan^{-1}x}{x^3} + \frac{84x\frac{\pi}{8}}{\sin\pi x}\right) =$$

$$\lim_{x\to 0}\frac{\left[x + \frac{1^2}{3!}x^3 + \ldots\right] - \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\right]}{x^3} + \frac{84}{8}$$

$$= \lim_{x\to 0}\frac{x^3\left(\frac{1}{3!} + \frac{1}{3}\right) + x^5\left(\ \right) + \ldots\ldots}{x^3} + \frac{21}{2} = 11$$

Q.10 [37]

$$\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}\,(bx - \sin x)} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{\sqrt{a+x}}\ \lim_{x\to 0}\frac{x^3}{bx - \sin x} = 1$$

$$\Rightarrow \frac{1}{\sqrt{a}} \cdot \lim_{x\to 0}\frac{x^3}{bx - \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\right]} = 1$$

$$\Rightarrow \frac{1}{\sqrt{a}} \cdot \lim_{x\to 0}\frac{x^3}{(b-1)x + \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots}$$

If limit exists, then $b - 1 = 0 \Rightarrow b = 1$

so $\frac{1}{\sqrt{a}} \cdot \lim\limits_{x\to 0}\dfrac{x^3}{x^3\left[\frac{1}{6} - \frac{x^2}{120} + \ldots\right]} = 1$

$$\Rightarrow \frac{1}{\sqrt{a}} \times \frac{1}{\frac{1}{6}} = 1 \Rightarrow a = 36$$

so $a = 36$, $b = 1$

Q.11 [20]

$$f(x) = \sum_{\lambda=1}^{n}\left(x - \frac{5}{\lambda}\right)\left(x - \frac{4}{\lambda+1}\right)$$

$$f(0) = \sum_{\lambda=1}^{n}\left(-\frac{5}{\lambda}\right)\left(-\frac{4}{\lambda+1}\right)$$

$$\Rightarrow f(0) = \sum_{\lambda=1}^{n}\left(\frac{20}{(\lambda)(\lambda+1)}\right)$$

$$\Rightarrow f(0) = 20\sum_{\lambda=1}^{n}\left(\frac{1}{\lambda} - \frac{1}{\lambda+1}\right)$$

$$\Rightarrow f(0) = 20\left(1 - \frac{1}{n+1}\right)$$

$$\Rightarrow f(0) = \frac{20n}{n+1}$$

Now $\lim\limits_{n\to\infty} f(0) = \lim\limits_{n\to\infty}\dfrac{20n}{n+1}$

$$= \lim_{n\to\infty}\frac{20}{1+\frac{1}{n}} = \frac{20}{1+0} = 20$$

Q.12 [12]

$$\text{LHL} = \lim_{h\to 0^+} f(0-h) = \lim_{h\to 0^+} f(-h) = \lim_{h\to 0^+} (-1)^{[h^2]} = 1$$

$$\text{RHL} = \lim_{h\to 0^+} f(0+h) = \lim_{h\to 0^+} f(h) = \lim_{h\to 0^+}\left(\lim_{n\to\infty}\frac{1}{1+h^n}\right) = 1$$

Q.13 [21]

$$\lim_{x\to 0}\frac{e^{-nx} + e^{nx} - 2\cos\frac{nx}{2} - kx^2}{(\sin x - \tan x)}$$

$$= \lim_{x\to 0} \frac{2\left[1+\frac{n^2x^2}{2!}+\frac{n^4x^4}{4!}+.....\right]-2\left[1-\frac{n^2x^2}{4.2!}+\frac{n^4x^4}{16.4!}-........\right]-kx^2}{\left(x-\frac{x^3}{3!}+........\right)-\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+.......\right)}$$

$$= \lim_{x\to 0} \frac{x^2\left(n^2+\frac{n^2}{4}-k\right)+x^4\left(\frac{2n^4}{4!}-\frac{2n^4}{16.4!}\right)}{x^3\left(-\frac{1}{3!}-\frac{1}{3}\right)+..........}$$

limit exists, if coff. of x^2 is zero.

$\Rightarrow \quad n^2 + \frac{n^2}{4} - k = 0 \quad \Rightarrow \quad 4k = 5n^2$

so the possible value match that is $n = 2$
$k = 5$

Q.14 [11]

$$\lim_{n\to\infty} \frac{\sum_{r=1}^{n} r^2(n-r+1)}{\sum_{r=1}^{n} r^3}$$

$$= \lim_{n\to\infty} \frac{\sum_{r=1}^{n}(n+1)r^2 - \sum_{r=1}^{n} r^3}{\sum_{r=1}^{n} r^3}$$

$$= \lim_{n\to\infty}\left(\frac{(n+1)\frac{(n)(n+1)(2n+1)}{6}}{\frac{n^2(n+1)^2}{4}}-1\right) = \frac{1/3}{1/4} - 1$$

$$= \frac{4}{3} - 1 = \frac{1}{3}$$

Q.15 [99]

$$\lim_{n\to\infty} \frac{n^{98}}{n^{x-1}\left({}^xC_1 - \frac{{}^xC_2}{2n} + \frac{{}^xC_3}{6n^2} - ...\right)} = \frac{1}{99}$$

the limit obviously exists if $x - 1 = 98$

Q.16 [1]

$$\because \quad (1+x)^{\frac{1}{x}} = e\left[1-\frac{x}{2}+\frac{11}{24}x^2 - .....\right]$$

Now $\ell = \lim_{x\to 0} \frac{e-(1+x)^{\frac{1}{x}}}{\tan x}$

$$= \lim_{x\to 0} \frac{e - e\left[1-\frac{x}{2}+\frac{11}{24}x^2-.....\right]}{x+\frac{x^3}{3}+\frac{2x^5}{15}+.....} = \frac{e}{2}$$

KVPY
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Q.1 (C)

$$\lim_{x\to\infty} x\sin(e^{-x})f(x) = \lim_{x\to\infty} \frac{\sin(e^{-x})}{e^{-x}}\frac{x}{e^{x}}f(x)$$
$$= 1 \times (0) \times M = 0$$

Q.2 (D)

$$\lim_{x\to 2} \frac{a^x + a^{3-x} - (a^2+a)}{a^{3-x} - a^{x/2}} \left(\frac{0}{0}\right)$$

Applying L hospital rule

$$\lim_{x\to 2} \frac{a^x \ell na - a^{3-x}\ell na}{-a^{3-x}\ell na - \frac{1}{2}a^{x/2}\ell na}$$

$$\frac{a^2\ell na - a\ell na}{-a\ell na - \frac{a}{2}\ell na} = \frac{a^2 - a}{-\frac{3a}{2}} = \frac{2}{3}(1-a)$$

Q.3 (A)

$$\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^{6/x^2} \qquad (1)^{\infty}$$

$$e^{\lim_{x\to 0}\frac{6}{x^2}\left(\frac{x}{\sin x}-1\right)}$$

$$e^{\lim_{x\to 0}\frac{6}{x^2}\left(\frac{x-\sin x}{\sin x}\right)}$$

$$e^{\lim_{x\to 0}\frac{6}{x^2}\left\{\frac{x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}....\right)}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}....\right)}\right\}}$$

$$e^{\lim_{x\to 0}\frac{6}{x^2}\frac{\left\{\frac{x^3}{3!}-\frac{5^5}{5!}.....\right\}}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}.....\right)}}$$

$$e^{\lim_{x\to 0}\frac{6x^3}{x^3}\frac{\left\{\frac{1}{3!}-\frac{x^2}{5!}.....\right\}}{\left\{1-\frac{x^2}{3!}....\right\}}} = e^1$$

Q.4 (D)

$$C(\theta) = \sum_{n=0}^{\infty}\frac{\cos(n\theta)}{n!}$$

$$= \lim_{n\to\infty}\left(1+\frac{\cos\theta}{1!}+\frac{\cos 2\theta}{2!}+\frac{\cos 3\theta}{3!}+...+\frac{\cos n\theta}{n!}\right)$$

$C(0) = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...$ up to ∞ term

= e

$C(\pi) = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ...$ up to ∞ term

Clearly C(0), C(π) = 1

$C(0) \,.\, C(\pi) = e + \frac{1}{e} > 2$

$C(0) = -\frac{\sin\theta}{2} - 2\frac{\sin 2\theta}{2!} - \frac{3\sin 3\theta}{3!} + ...$ up to $\propto$ term

And that value is equal to zero at 0 = 0

Q.5 (A)

$$\lim_{x\to\infty} \frac{x^2 \int_0^x e^{t^3} dt}{e^{x^3}}$$

Apply L Hospital

$$\lim_{x\to\infty} \frac{2x\int_0^x e^{t^3} + x^2 e^{x^3}}{3x^2 e^{x^3}}$$

$$\lim_{x\to\infty} \frac{2\int_0^x e^{t^3} + xe^{x^3}}{3x^2 e^{x^3}}$$

$$\lim_{x\to\infty} \frac{2e^{x^3} + e^{x^3} + 3x^3 e^{x^3}}{3e^{x^3} + 9x^3 e^{x^3}}$$

$$\lim_{x\to\infty} \frac{3+3x^3}{3+9x^3} = \frac{1}{3}$$

Q.6 (D)

Rationalise

$$\lim_{x\to-\infty} \left((\sqrt{4x^2 - x} + 2x) \times \frac{\sqrt{4x^2 - x} - 2x}{\sqrt{4x^2 - x} - 2x} \right)$$

$$\lim_{x\to-\infty} \left(\frac{-x}{|x|\sqrt{4 - \frac{1}{x}} - 2x} \right) \text{at } x \to -\infty, x| = -x$$

$$\lim_{x\to-\infty} \left(\frac{-x}{-x\sqrt{4 - \frac{1}{x}} - 2x} \right) = \frac{1}{2+2} = \frac{1}{4}$$

Q.7 (B)

$$\lim_{n\to\infty} (3^n)^{1/2n} = \left(\left(\frac{2}{3}\right)^n + 1 \right)^{1/2n}$$

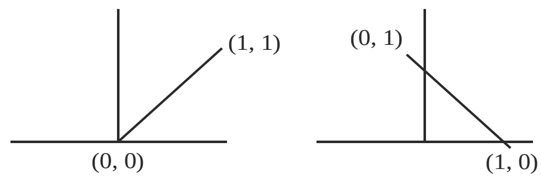
Put $\lim_{n\to\infty} \sqrt{3}$

Q.8 (B)

$| f (x) = f (x) | = | x - y| \quad \Rightarrow \frac{|f(x) - f(y)|}{|x - y|} = 1$

Take $\lim_{x\to y} \left| \frac{f(x) - f(y)}{x - y} \right| = 1$

$\Rightarrow \lim_{x\to y} |f'(x)| = 1 \quad \Rightarrow f'(x) = \pm 1$



Hence two function possible

Q.9 (A)

(I) $\lim_{n\to\infty} \left(\frac{2^n}{2^n} + \left(\frac{-2}{2}\right)^n \right)$

$= \lim_{n\to\infty} (1 + (-1)^n)$ does not exist

(II) $\lim_{n\to\infty} \left(\left(\frac{3}{4}\right)^n + \left(\frac{-3}{4}\right)^n \right) = 0 + 0 = 0.$

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (4)

$$\lim_{x\to 0} \frac{ax - (e^{4x} - 1)}{ax.4x}$$

Apply L' Hospital

$\lim_{x\to 0} \frac{a - (e^{4x}) \cdot 4}{8ax} \qquad \left(\frac{a - 4}{0} \text{ form} \right)$

4 limit exist a = 4

$\lim_{x\to 0} \frac{4 - 4e^{4x}}{32x} = \lim_{x\to 0} \frac{1 - e^{4x}}{8x} = \frac{-1}{2}$

a = 4 , b = $\frac{-1}{2}$

2 (a + b) = $2\left(4 - \frac{1}{2}\right) = 7$

Q.2 (1)

$$= \lim_{x\to 0} \frac{2\left[\sin\left(\frac{\pi}{6}+x-\frac{\pi}{6}\right)\right]}{\sqrt{3}x(\sqrt{3})} = \lim_{x\to 0} \frac{4}{3}\frac{\sin x}{x} = \frac{4}{3}$$

Q.3 (1)
By L–H rule

$$L = \lim_{x\to a} \frac{f(a)-af'(x)}{1}$$

$\therefore$ L=4–a(2)

Q.4 [4]

$$\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$$

$$\Rightarrow \lim_{x\to 0} \frac{a\left(1+x+\frac{x^2}{2!}...\right) \quad b\left(1 \quad \frac{x^2}{2!}+...\right)+c\left(1 \quad x+\frac{x^2}{2!}\right)}{\left(\frac{x\sin x}{x}\right)x} = 2$$

a – b + c = 0 .....(1)
a – c = 0 .....(2)

$$\& \quad \frac{a+b+c}{2} = 2$$

$$\Rightarrow \boxed{a+b+c=4}$$

Q.5 **(4)**

$$\lim_{x\to 0^+} \frac{\cos^{-1}x}{(1-x^2)} \times \frac{\sin^{-1}x}{x} = \frac{\pi}{2}$$

Q.6 **(1)**
We know that
$r \le [r] < r + 1$
and $2r \le [2r] < 2r + 1$
$3r \le [3r] < 3r + 1$
M M M
$nr \le [nr] < nr + 1$

r + 2r + ...+ nr
$\le$ [r] + [2r] +....+ [nr] < (r + 2r +....+ nr) + n

$$\frac{\frac{n(n+1)}{2}\cdot r}{n^2} \le \frac{[r]+[2r]+......+[nr]}{n^2} < \frac{\frac{n(n+1)}{2}r+n}{n^2}$$

Now,

$$\lim_{n\to\infty} \frac{n(n+1)\cdot r}{2\cdot n^2} = \frac{r}{2}$$

$$\text{and} \lim_{n\to\infty} \frac{\frac{n(n+1)r}{2}+n}{n^2} = \frac{r}{2}$$

So, by Sandwich Theorem, we can conclude that

$$\lim_{n\to\infty} \frac{[r]+[2r]+......+[nr]}{n^2} = \frac{r}{2}$$

Q.7 **(1)**

$$\lim_{\theta\to 0} \frac{\tan\left(\pi\left(1-\sin^2\theta\right)\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$= \lim_{\theta\to 0} \frac{-\tan\left(\pi\sin^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$= \lim_{\theta\to 0} -\left(\frac{\tan\left(\pi\sin^2\theta\right)}{\left(\pi\sin^2\theta\right)}\right)\left(\frac{2\pi\sin^2\theta}{\sin\ 2\pi\sin^2\theta}\right)\times\frac{1}{2}$$

$= \frac{-1}{2}$ **Option (1)**

Q.8 **(4)**

$$\lim_{x\to 0} \frac{\left(x+\frac{x^3}{3!}...\right)-\left(x-\frac{x^3}{3}...\right)}{3x^3} = \frac{1}{6}$$

So 6L + 1 = 2
So 6L + 1 = 2

Q.9 **(3)**
If f(x + y) = f(x).f(y) & f'(0) = 3 then
f(x) = ax $\Rightarrow$ f'(x) = ax.λna
$\Rightarrow$ f'(0) = λna = 3 $\Rightarrow$ a = e3
$\Rightarrow$ f(x) = (e3)x = e3x

$$\lim_{x\to 0} \frac{f(x)-1}{x} = \lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\times 3\right) = 1\times 3 = 3$$

Q.10 (4)

Q.11 (3)

Q.12 [3]

Q.13 **[3]**

Q.14 [7]

Q.15 **(4)**

Q.16 (3)

Q.17 (1)

Q.18 (1)

Q.19 **(4)**

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 **(D)**

$$\lim_{x\to 0} e^{\frac{x\ell n(1+b^2)}{x}} = 1 + b^2 = 2b\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{2}\left(b+\frac{1}{b}\right)$$

We know $b + \frac{1}{b} \ge 2$

$\Rightarrow \sin^2\theta \ge 1$ but $\sin^2\theta \le 1 \Rightarrow \sin^2\theta = 1 \Rightarrow \theta = \pm\frac{\pi}{2}$

Q.2 **(B)**

$$\ell im_{x\to\infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$$

$$\ell im_{x\to\infty}\left(\frac{x^2(1-a)+x(1-a-b)+(1-b)}{x+1}\right) = 4$$

Limit is finite

It exists when $1 - a = 0 \Rightarrow a = 1$

then $$\ell im_{x\to\infty}\left(\frac{1-a-b+\frac{1-b}{x}}{1+\frac{1}{x}}\right) = 4$$

$\therefore 1 - a - b = 4 \Rightarrow b = -4$

Q.3 **(B)**

$((1+a)^{1/3} - 1)x^2 + ((a+1)^{1/2} - 1)x + ((a+1)^{1/6} - 1) = 0$

let $a + 1 = t^6$

$\therefore$ $(t^2 - 1)x^2 + (t^3 - 1)x + (t - 1) = 0$

$(t + 1)x^2 + (t^2 + t + 1)x + 1 = 0$

As $a \to 0$, $t \to 1$

$2x^2 + 3x + 1 = 0 \Rightarrow x = -1$ and $x = -\frac{1}{2}$

Q.4 **[0]**

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{x+\sqrt{x}} = \frac{1}{4}$$

Hence $$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{(x-1)+\sin(x-1)}\right\}^{1+\sqrt{x}} = \frac{1}{4}$$

put $x = 1 + h$,

$$\lim_{h\to 0}\left\{\frac{-ah+\sin h}{h+\sin h}\right\}^{1+\sqrt{1+h}} = \frac{1}{4}$$

or $\frac{-a+1}{2} = \frac{1}{2}$ or $-\frac{1}{2}$ $\Rightarrow a = 0$ or 2

But at a = 2, $\frac{-ah+\sin h}{h+\sin h}$ tends to negative value

So correct Answer is a = 0

However a = 2 may be accepted if this is not considered

Q.5 **(A,B,C)**

$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

Let $\frac{\pi}{2}\sin x = \theta$ $\quad\because \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore$ $f(x) = \sin\left(\frac{\pi}{6}\sin\theta\right)$

Let $\frac{\pi}{6}\sin\theta = \phi$ $\quad\therefore \phi \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

$\therefore$ $f(x) = \sin\phi \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\therefore$ **(A)**

Now $fog(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$

Clearly, range of fog is also $\left(-\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ **(B)**

Now, $$\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x}$$

$$= \lim_{x\to 0}\frac{2}{\pi}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)} \times \frac{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\sin x}{x}\times x}$$

$$= \lim_{x\to 0}\frac{2}{\pi}\times\frac{\pi}{6}\times\frac{\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x}\times\frac{\frac{\pi}{2}\sin x}{x}$$

$\frac{1}{3}\times\frac{\pi}{2}=\frac{\pi}{6}$

$\therefore$ **(C)**

Now, gof(x) = 1

$\Rightarrow \frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right) = 1$

$\Rightarrow \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)=\frac{2}{\lambda}\cong\frac{2}{3.14}>\frac{1}{2}$

$\therefore$ **(D)**

Q.6 **[2]**

$\because$ m ≥ 2 and n ≥ 2

$= \lim_{a\to 0}\frac{e(e^{\cos(a^n)-1}-1)}{(\cos(a^n)-1)}\times\left(\frac{\cos(a^n)-1}{(a^n)^2}\right)\frac{a^{2n}}{a^m}$

=

$e\times\lim_{a\to 0}\left(\frac{e^{\cos(a^n)-1}-1)}{\cos(a^n)-1)}\right)\times\lim_{a\to 0}\left(\frac{\cos(a^n)-1}{a^{2n}}\right)\times\lim_{a\to 0}a^{2n-m}$

$= e \times 1 \times -\frac{1}{2}\times\lim_{a\to 0}a^{2n-m}$

Now $\lim_{a\to 0} a^{2n-m}$ must be equal to 1.

i.e. 2n – m = 0

$\frac{m}{n}=2$

Q.7 [7]

$\lim_{x\to 0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$

$\lim_{x\to 0}\frac{x^3\beta}{\alpha x-\sin x}=1$

$\lim_{x\to 0}\frac{3x^2\beta}{\alpha-\cos x}=1$

$\therefore \alpha-1=0 \qquad \Rightarrow \alpha=1$

$\lim_{x\to 0}\frac{6x\beta}{\sin x}=1 \Rightarrow 6\beta=1$

$\therefore 6(\alpha+\beta)=7$

Q.8 **(CD)**

$f(1^+)=\lim_{h\to 0}\frac{1-(1+h)(1+h)}{h}\cos\frac{1}{h}$

$=\lim_{h\to 0}\frac{1-(1+h)^2}{h}\cos\frac{1}{h}$

$=\lim_{h\to 0}\frac{-h^2-2h}{h}\cos\frac{1}{h}$

$=\lim_{h\to 0}(-h-2)\cos\frac{1}{h}$

$\Rightarrow \lim_{h\to 0} f(1^+)$ does not exist

$f(1^-)=\lim_{h\to 0}\frac{1-(1-h)(1+h)}{h}\cos\frac{1}{h}$

$=\lim_{h\to 0}\frac{1-(1-h)^2}{h}\cos\frac{1}{h}$

$=\lim_{h\to 0}\frac{h^2}{h}\cos\frac{1}{h}$

$=\lim_{h\to 0}h\cos\frac{1}{h}=0$

Q.9 (B, D)

P-1 :

$\lim_{h\to 0}\frac{f(h)-f(0)}{\sqrt{|h|}}$ = exist and finite

(B) $f(x) = x^{2/3}$, $\lim_{h\to 0}\frac{h^{2/3}-(0)}{\sqrt{|h|}}=\lim_{h\to 0}\frac{|h|^{2/3}}{\sqrt{|h|}}=0$

(D) $f(x) = |x|$, $\lim_{h\to 0}\frac{|h|-0}{\sqrt{|h|}}\Rightarrow\lim_{h\to 0}\sqrt{|h|}=0$

P-2 :

$\lim_{h\to 0}\frac{f(h)-f(0)}{\sqrt{|h|}}$ = exist and finite

(A) $f(x) = x|x|$, $\lim_{h\to 0}\frac{h(h)-0}{h^2}=\left[\begin{array}{l}RHL=\lim_{x\to\infty}\frac{h^2}{h^2}=1\\ LHL=\lim_{x\to\infty}\frac{-h^2}{h^2}=-1\end{array}\right.$

(C) $f(x) = \sin x$ $\lim_{x\to\infty}\frac{\sinh-0}{h^2}=DNE$

Q.10 [1.00]

$\lim_{x\to 0^+}\frac{e^{\left(\frac{\ell n(1-x)}{x}\right)}-\frac{1}{e}}{x^a}$

$$= \lim_{x\to 0^+} \frac{1}{e} \frac{e^{\left(1+\frac{\ell n(1-x)}{x}\right)} - 1}{x^a}$$

$$= \frac{1}{e} \lim_{x-0^+} \frac{1+\frac{\ell n(1-x)}{x}}{x^a}$$

$$= \frac{1}{e} \lim_{x-0^+} \frac{\ell n(1-x)+x}{x^{(a+1)}}$$

$$= \frac{1}{e} \lim_{x-0^+} \frac{\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-....\right)+x}{x^{a+1}}$$

Thus, a=1

Q.11 [8]

$$\lim_{x\to\frac{\pi}{2}} \frac{4\sqrt{2}\cdot 2\sin 2x\cos x}{2\sin 2x\sin\frac{3x}{2}+\left(\cos\frac{5x}{2}-\cos\frac{3x}{2}\right)-\sqrt{2}(1+\cos 2x)}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x\cos^2 x}{2\sin 2x\left(\sin\frac{3x}{2}-\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x\cos^2 x}{4\sin x\cos x\left(2\cos x\cdot\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x}{8\sin x\cdot\sin\frac{x}{2}-2\sqrt{2}} = 8$$

Continuity and Derivability

EXERCISES

ELEMENTRY

Q.1 (2)

$f(\pi/2) = 3$. Since f(x) is continuous at $x = \pi/2$

$$\Rightarrow \lim_{x\to\pi/2}\left(\frac{k\cos x}{\pi - 2x}\right) = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6.$$

Q.2 (1)

Here $\lim_{x\to 0+} f(x) = k,\ \lim_{x\to 0-} f(x) = -k$ and $f(0) = k$

But f(x) is continuous at x = 0, therefore k must be zero.

Q.3 (3) $f(0) = 0$

$\lim_{x\to 0-} f(x) = \lim_{h\to 0} e^{-1/h} = 0$ and

$\lim_{x\to 0+} f(x) = \lim_{h\to 0} e^{1/h} = \infty$

Hence function is discontinuous at x = 0

Q.4 (2)

$$f(x) = \frac{x+1}{(x-3)(x+4)}.$$

Hence the points are 3, – 4.

Q.5 (3)

$\lim_{x\to 0^+} f(x) = x^2 \sin\frac{1}{x}$, but $-1 \le \sin\frac{1}{x} \le 1$ and $x \to 0$

Therefore, $\lim_{x\to 0^+} f(x) = 0 = \lim_{x\to 0^-} f(x) = f(0)$

Hence f(x) is continuous at x = 0.

Q.6 (4)

$f(0-) = \lim_{x\to 0-} k(2x - x^2) = 0$;

$f(0+) = \lim_{x\to 0+} \cos x = 1$

$\therefore\ f(0) = \cos x = 1$

Hence no value of k can make $f(0-) = 1$.

Q.7 (2)

$\lim_{x\to 1} f(x) = \lim_{x\to 1} x + 1 = 2 = k.$

Q.8 (2)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{2\sin 2x}{2x.5} = \frac{2}{5} = k.$$

Q.9 (3)

$\lim_{x\to 1+} f(x) = 0$ and $\lim_{x\to 1+-} f(x)1 + 1 = 2$

Hence f(x) is discontinuous at x= 1.

Q.10 (2)

$\lim_{x\to a-} f(x) = -1,\ \lim_{x\to a+} f(x) = 1,\ f(a) = 1.$

Q.11 (1)

$\lim_{x\to 2-} f(x) = 3,\ \lim_{x\to 2+} f(x) = 3$ and f(2) = 3

Q.12 (3)

Here $f\left(\frac{3\pi}{4}\right) = 1$ and $\lim_{x\to 3\pi/4-} f(x) = 1$

$$\lim_{x\to 3\pi/4+} f(x) = \lim_{h\to 0} 2\sin\frac{2}{9}\left(\frac{3\pi}{4} + h\right) = 2\sin\frac{\pi}{6} = 1.$$

Hence $f(x)$ is continuous at $x = \frac{3\pi}{4}$.

Q.13 (2)

$\lim_{x\to 0-} f(x) = 1 + 1 = 2,\ \lim_{x\to 0+} f(x) = 0,\ f(0) = 2.$

Q.14 (2)

$\lim_{x\to 2} f(x) = \lim_{x\to 2} (x+2)(x^2+4) = 32,\ f(2) = 16.$

Q.15 (2)

$$\lim_{x\to -5} f(x) = \frac{(x-2)(x+5)}{(x+5)(x-3)} = \frac{-7}{-8} = \frac{7}{8}.$$

Q.16 (4)

By definition of continuity, we know that

$\lim_{x\to 3+} f(x) = f(3) = \lim_{x\to 3-} f(x)$

$\Rightarrow \lim_{x\to 3-} f(x) = 4$ or $\lim_{h\to 0} 3 - h + \lambda = 4$

$\Rightarrow 3 + \lambda = 4 \Rightarrow \lambda = 1$

Q.17 (1)

$\lim_{x\to 0} (\cos x)^{1/x} = k$

$$\Rightarrow \lim_{x\to 0}\frac{1}{x}\log(\cos x) = \log k$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{x}\lim_{x\to 0}\log\cos x = \log k$$

$$\Rightarrow \lim_{x\to 0} \frac{1}{x} \times 0 = \log_e k \Rightarrow k = 1 .$$

Q.18 (3)

f(x) is continuous at $x = \frac{\pi}{2}$, then

$$\lim_{x\to \pi/2} f(x) = f(0)$$

or $\lambda = \lim_{x\to \pi/2} \frac{1-\sin x}{\pi - 2x}$, $\left(\frac{0}{0}\text{form}\right)$

Applying L-Hospital's rule,

$$\lambda = \lim_{x\to \pi/2} \frac{-\cos x}{-2} \Rightarrow \lambda = \lim_{x\to \pi/2} \frac{\cos x}{2} = 0.$$

Q.19 (2)

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3}(x+3) = 6$$

and $f(3) = 2(3) + k = 6 + k$

$\because f$ is continuous at x = 3; $\therefore$ 6 + k = 6

$\Rightarrow$ k = 0.

Q.20 (3)

$$\text{L.H.L.} = \lim_{x\to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} = k$$

$$\text{R.H.L.} = \lim_{x\to 0^+} (2x^2 + 3x - 2) = -2$$

Since it is continuous, L.H.L = R.H.L $\Rightarrow$ k = –2.

Q.21 (1)

$$f(x) = \begin{cases} \frac{1-\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$ continuous at x = 0

$$\lim_{x\to 0^+} f(x) = f(0) \Rightarrow \lim_{x\to 0} \frac{2.\sin^2 x/2}{x} = k$$

$$\Rightarrow \lim_{x\to 0} \frac{2\sin^2 x/2}{\left(\frac{x}{2}\right)^2} . \frac{x}{4} = k \Rightarrow k = 0.$$

Q.22 (4)

Given $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\Rightarrow \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} = \frac{e^{\infty}-1}{e^{\infty}+1} = -1$$

$$\Rightarrow \lim_{x\to 0^-} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} = \frac{1-e^{-\frac{1}{x}}}{1+e^{\frac{1}{x}}} = \frac{1-e^{-\infty}}{1+e^{\infty}} = 1$$

So, $\lim_{x\to 0} f(x)$ exists at x = 0, but at it is not continuous.

Q.23 (3)

$$f(x) = \frac{2x^2+7}{x^2(x+3) - 1(x+3)} \frac{9x^2+7}{(x^2-1)(x+3)}$$

$$= \frac{2x^2+7}{(x-1)(x+1)(x+3)}$$

Hence points of discontinuity are x =1, x = –1 and x = –3 only.

Q.24 (1)

$$f(5) = \lim_{x\to 5} f(x) = \lim_{x\to 5} \frac{x^2 - 10x + 25}{x^2 - 7x + 10}$$

$$= \lim_{x\to 5} \frac{(x-5)^2}{(x-2)(x-5)} = \frac{5-5}{5-2} = 0 .$$

Q.25 (3)

For continuity at x = 0, we must have

$$f(0) = \lim_{x\to 0} f(x)$$

$$= \lim_{x\to} (x+1)^{\cot x}$$

$$= \lim_{x\to 0} \left\{(1+x)^{\frac{1}{x}}\right\}^{x \cot x}$$

$$= \lim_{x\to 0} \left\{(1+x)^{\frac{1}{x}}\right\}^{\lim_{x\to 0}\left(\frac{x}{\tan x}\right)} = e^1 = e$$

Q.26 (1)

It is obvious that |x| is continuous for all x.

Now, $Rf'(x) = \lim_{h\to 0} \frac{|0+h| - 0}{h} = 1$

$$Lf'(x) = \lim_{h\to 0} \frac{|0-h| - 0}{-h} = -1$$

Hence f(x) = |x| is not differentiable at x = 0.

Q.27 (1)

$f(x) = x^p \sin\frac{1}{x}, x \neq 0$ and $f(x) = 0, x = 0$

Since at x = 0, f(x) is a continuous function

$\therefore \lim_{x\to 0} f(x) = f(0) = 0$

$\Rightarrow \lim_{x\to 0} x^p \sin\frac{1}{x} = 0 \Rightarrow p > 0$.

is differentiable at , if exists

$\Rightarrow \lim_{x\to 0} \frac{x^p \sin\frac{1}{x} - 0}{x - 0}$ exists

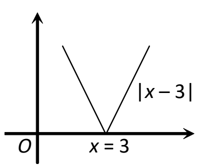
$\Rightarrow \lim_{x\to 0} x^{p-1} \sin\frac{1}{x}$ exists

$\Rightarrow$ p – 1 > 0 or p > 1

If $p \leq 1$, then $\lim_{x\to 0} x^{p-1} \sin\left(\frac{1}{x}\right)$ does not exist and at x = 0 f(x) is not differentiable.

$\therefore$ for $0 < p \leq 1$ f(x) is a continuous function at x = 0 but not differentiable.

Q.28 (4)



$\lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0} |3-h-3| = 0$

$\lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0} |3+h-3| = 0$

$\because \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

Hence f is continuous at x = 3

Now $Lf'(3) = \lim_{h\to 0} \frac{f(3-h)-f(3)}{-h}$

$= \lim_{h\to 0} \frac{|3-h-3|-0}{-h} = \lim_{h\to 0} \frac{h}{-h} = -1$

$Rf'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{|3+h-3|-0}{h} = 1$

$\because Lf'(3) \neq Rf'(3)$. Hence f is not differentiable at x = 3.

Q.29 (2)

We have $Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{\{(1+h)^3 - 1\} - 0}{h} = 3$

$Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{\{(1-h)-1\}-0}{-h} = 1$

$\therefore Rf'(1) \neq Lf'(1) \Rightarrow f(x)$ is not differentiable at x =1 $x = 1$.

Now, $f(1+0) = \lim_{h\to 0} f(1+h) = 0$

and $f(1-0) = \lim_{h\to 0} f(1-h) = 0$

$\therefore$ $f(1+0) = f(1-0) = f(0)$ is continuous at

Hence at is continuous and not differentiable.

Q.30 (3)

We have, $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ e^{x}, & x < 0 \end{cases}$

Clearly, *f*(x) is continuous and differentiable for all non zero x.

Now

$\lim_{x\to 0-} f(x) = \lim_{x\to 0} e^x = 1, \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} f(x)e^{-x} = 1$

Also, $f(0) = e^0 = 1$. So, f(x) is continuous for all x.

(LHD at x = 0) $= \left(\frac{d}{dx}(e^x)\right)_{x=0} = 1$

(RHD at x = 0) $= \left(\frac{d}{dx}\left(e^{-x}\right)\right) = -1$

So, f(x) is not differentiable at x = 0.

Hence $f(x) = e^{-|x|}$ is everywhere continuous but not differentiable at .

Q.31 (3)

$\lim_{h\to 0^-} 1 + (2-h) = 3$,

$\lim_{h\to 0^+} 5 - (2+h) = 3$, $f(2) = 3$

Hence, f is continuous at x = 2

Now $Rf'(x) = \lim_{h\to 0} \frac{5-(2+h)-3}{h} = -1$

$Lf'(x) = \lim_{h\to 0} \frac{1+(2-h)-3}{-h} = 1$

$\because Rf'(x) \neq Lf'(x)$;

$\therefore$ f is not differentiable at x = 2

Q.32 (3)

$$f(x)=\begin{cases}0, & x<0\\ x^2, & x\ge 0\end{cases};\ \lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)=0$$

and $\lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)=\lim_{h\to 0}(0+h)^2=0$

$\Rightarrow \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$

Hence f(x) is continuous function at x = 0.

$$Lf'(x)=\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim_{h\to 0}\frac{f(0-h)-0}{-h}=\lim_{h\to 0}\frac{0-0}{-h}=0$$

$$Rf'(x)=\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{(0+h)^2-0}{h}=0$$

$\Rightarrow Lf'(x)=Rf'(x)$

Hence f(x) is differentiable at x =0

Now $f'(x)=\begin{cases}0\ , & x<0\\ 2x\ , & x\ge 0\end{cases};$

$\lim_{x\to 0^-} f'(x)=\lim_{h\to 0} f'(0-h)=0$

and $\lim_{x\to 0^+} f'(x)=\lim_{h\to 0} f'(0+h)=\lim_{h\to 0} 2(0+h)=0$

$\Rightarrow \lim_{x\to 0^-} f'(x)=\lim_{x\to 0^+} f'(x)=0$

Hence f(x) is continuous function at x = 0.

Now

$$Lf''(x)=\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$=\lim_{h\to 0}\frac{0-0}{-h}=0$$

$$Rf''(x)=\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{2(0+h)-0}{h}=\lim_{h\to 0}\frac{2h}{h}=2 \Rightarrow$$

$Lf''(x)\ne Rf''(x)$

Hence $f'(x)$ is not differentiable at x = 0.

Q.33 (4)

$\lim_{x\to 0} f(x)=x^2\sin\left(\frac{1}{x}\right)$, but $-1\le \sin\left(\frac{1}{x}\right)\le 1$ and x → 0

$\lim_{x\to 0^+} f(x)=0=\lim_{x\to 0^-} f(x)=f(0)$

Therefore f(x) is continuous at x = 0. Also, the function $f(x)=x^2\sin\frac{1}{x}$ is differentiable because

$$Rf'(x)=\lim_{h\to 0}\frac{h^2\sin\frac{1}{h}-0}{h}=0,$$

$$Lf'(x)=\lim_{h\to 0}\frac{h^2\sin(1/-h)}{-h}=0.$$

Q.34 (2)

By definition, $f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$$=\lim_{h\to 0}\frac{\frac{1}{2(1+h)-5}-\left(\frac{-1}{3}\right)}{h}=\lim_{h\to 0}\frac{\left(\frac{1}{2h-3}+\frac{1}{3}\right)}{h}$$

$$=\lim_{h\to 0}\left(\frac{3+2h-3}{3h(2h-3)}\right)=\lim_{h\to 0}\left(\frac{2h}{3h(2h-3)}\right)$$

$$=\lim_{h\to 0}\frac{2}{3(2h-3)}=\frac{2}{3(-3)}=\frac{-2}{9}.$$

Q.35 (4)

Since function $|x|$ is not differentiable at $x=0$

$\therefore |x^2-3x+2|=|(x-1)(x-2)|$

Hence is not differentiable atn x = 1 and 2

Now $f(x)=(x^2-1)|x^2-3x+2|\cos(|x|)$ is not differentiable at x = 2

For $1<x<2$,

$f(x)=-(x^2-1)(x^2-3x+2)+\cos x$

For $2<x<3$,

$f(x)=+(x^2-1)(x^2-3x+2)+\cos x$

$Lf'(x)=-(x^2-1)(2x-3)-2x(x^2-3x+2)-\sin x$

$Lf'(2)=-3-\sin 2$

$Rf'(x)=(x^2-1)(2x-3)+2x(x^2-3x+2)-\sin x$

$Rf'(2)=(4-1)(4-3)+0-\sin 2=3-\sin 2$

Hence $Lf'(2)\ne Rf'(2)$.

Q.36 (3)

$$f(x)=\begin{cases}x, & 0\le x\le 1\\ 2x-1, & x>1\end{cases}$$

$$\lim_{x\to1^-} f(x) = \lim_{h\to0} f(1-h) = \lim_{h\to0}(1-h) = 1$$

$$\lim_{x\to1^+} f(x) = \lim_{h\to0} f(1+h) = \lim_{h\to0} 2(1+h)-1 = 1$$

$$\because \lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = 1$$

$\therefore$ Function is continuous at x =1.

$$Lf'(1) = \lim_{h\to0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to0}\frac{(1-h)-1}{-h} = 1$$

$$Rf'(1) = \lim_{h\to0}\frac{f(1+h)-f(1)}{-h} = \lim_{h\to0}\frac{2+2h-1-1}{h} = 2$$

$\therefore$ $Lf'(1) \neq Rf'(1)$

$\therefore$ Function is not differentiation at x =1

Q.37 (3)

Since the function is defined for $x \geq 0$ i.e. not defined for x< 0. Hence the function neither continuous nor differentiable at x = 0.

Q.38 (1)

$f'(1) = \lim_{h\to0}\frac{f(1+h)-f(1)}{h}$; As function is differentiable so it is continous as it is given that $\lim_{h\to0}\frac{f(1+h)}{h} = 5$ and hence f(1) = 0. Hence

$$f'(1) = \lim_{h\to0}\frac{f(1+h)}{h} = 5.$$

Q.39 (2)

$$f(1) = f\left(\frac{1}{2}\right) = f\left(\frac{1}{3}\right) = ... = \lim_{n\to\infty} f\left(\frac{1}{n}\right) = 0$$

Since there are infinitely many points in $x \in (0,1)$

where f(x) =0 and $\lim_{n\to\infty} f\left(\frac{1}{n}\right) = 0 \Rightarrow f(0) = 0$

And since there are infinitely many points in the neighbourhood of x = 0 such that

$\Rightarrow$ f(x) remains constant in the neighbourhood of x = 0 $\Rightarrow$ $f'(0) = 0$

JEE-MAIN

OBJECTIVE QUESTIONS

Q.1 **(1)**

$$\lim_{x\to0} f(x) = f(0)$$

$$\lim_{x\to0}\frac{\cos(\sin x)-\cos x}{x^2} = a$$

$$\Rightarrow \frac{2}{x^2}\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right) = a$$

$$\Rightarrow a = \lim_{x\to0} 2.\frac{\sin\left(\frac{\sin x + x}{2}\right)}{\frac{\sin x + x}{2}}.\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\frac{x-\sin x}{2}}.$$

$$\frac{1}{4}\left(\frac{\sin x + x}{x}\right)\left(\frac{x-\sin x}{x}\right)$$

$$= 2.1\ .\ 1.\ \frac{1}{4}(1+1)(1-1) = 0$$

Q.2 **(2)**

$$f(x) = \begin{cases}\dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1 \leq x \leq 0\\[2mm] \dfrac{2x+1}{x+2}, & 0 \leq x \leq 1\end{cases}$$

since it is cont, so,

$$\lim_{x\to0^-} f(x) = f(0)$$

$$\lim_{h\to0}\frac{\sqrt{1+p(-h)}-\sqrt{1-p(-h)}}{-h} = -\frac{1}{2}$$

$$\lim_{h\to0}\frac{(1-ph)-(1+ph)}{-h\left\{\sqrt{1-ph}+\sqrt{1+ph}\right\}} = -\frac{1}{2}$$

$$\frac{+2p}{2} = -\frac{1}{2}$$

$$p = -1/2$$

Q.3 **(4)**

$$f(x) = \left|\left(x+\frac{1}{x}\right)[x]\right|, \quad x \in [-2, 2]$$

$$\lim_{x\to2^-} f(x) = \lim_{h\to0}\left|\left(\frac{5}{2}-h\right)1\right| = \frac{5}{2}$$

$$f(2) = \left|\frac{5}{2}\times2\right| = 5$$

so, discontinuous at x = 2

now defining function

$$f(x) = \left|\left(x+\frac{1}{2}\right)[x]\right| = \begin{cases}3 & ; -2 \leq x < -1\\ \frac{1}{2} & ; -1 \leq x < 0\\ 0 & ; 0 \leq x < 1\\ \frac{3}{2} & ; 1 \leq x \leq 2\\ 2 & ; 2 \leq x < 3\end{cases}$$

by defining the function we can say that this it is discontinuous at x = 0

Q.4 **(3)**

$$y = \frac{1}{t^2+t-2},\ t = \frac{1}{x-1}$$

$$y = \frac{1}{\frac{1}{(x-1)^2}+\frac{1}{x-1}-2}$$

$$y = \frac{(x-1)^2}{1+(x-1)-2(x-1)^2}$$

$$y = \frac{x^2-2x+1}{x-2x^2-2+4x} = \frac{x^2-2x+1}{-2x^2+5x-2}$$

$$y = \frac{(x-1)^2}{-2x^2+4x+x-2} = \frac{(x-1)^2}{-2x(x-2)+1(x-2)}$$

$$y = \frac{(x-1)^2}{(x-2)(-2x+1)}$$

by $\Rightarrow x \in R - \left\{2, +\frac{1}{2}\right\}$ so disc. at 1/2 & 2 let we also include x = 1 because at x = 1 't' is not defined.

Q.5 **(2)**

$$f(x) = \lim_{t\to\infty}\left\{\frac{(1+\sin\pi x)^t-1}{(1+\sin\pi x)^t+1}\right\}$$

$$\lim_{t\to 0}\frac{(1+\sin\pi x)^t-1}{(1+\sin\pi x)^t+1}\begin{cases} 0 & \begin{array}{c}1+\sin\pi x = 1\\ \sin\pi x = 0\\ \pi x = n\pi\\ x = n, n = 0,1,2,......\end{array}\\ -1 & \begin{array}{c}1+\sin\pi x > 1\\ \sin\pi x > \sin 0\\ x > n\end{array}\\ -1 & \begin{array}{c}1 > 1+\sin\pi x > 0\\ \pi x > -\frac{\pi}{2}\\ 0 > x > -\frac{n}{2}\end{array}\end{cases}$$

$$\text{Now } f(x) = \begin{cases} 0 & x = n, n = 1,2,3...\\ -1 & x > n\\ -1 & -\frac{n}{2} < n < 0\end{cases}$$

(i) $f(0^+) = -1$ (ii) $f(1^+) = -1$
$f(0^+) = -1$ $f(1^-) = -1$
$f(0) = 0$ $f(1) = 0$

Similarly for all integer the function will be discontinuous

Q.6 **(4)**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{1}{x}-\frac{2}{e^{2x}-1}\right)$$

$$= \lim_{x\to 0}\left(\frac{e^{2x}-1-2x}{x(e^{2x}-1)}\right) \text{ using L - hospital rule}$$

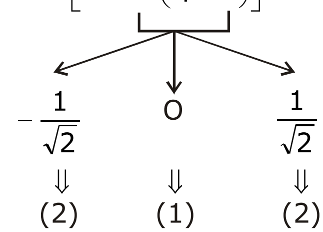
$$= \lim_{x\to 0}\left(\frac{e^{2x}-1-2x}{x(e^{2x}-1)}\right) = 1$$

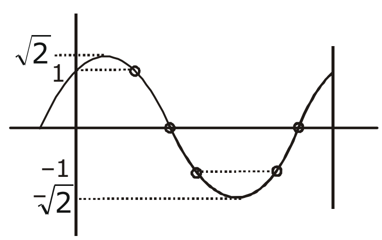
Q.7 **(4)**

$f(x) = |x-1| + |x-2| + \cos x$
All three functions are cont. in [0, 4] so sum of all these functions is also a cont. funs.

Q.8 **(2)**

$$f(x) = \left[\sqrt{2}\sin\left(\frac{\pi}{4}+h\right)\right]$$





Total solutions = 5

Q.9 **(4)**

$$f(1^+) = \lim_{x\to 1^+} x^2\left[\frac{1}{x^2}\right] = 0$$

$$f(1^-) = \lim_{x\to 1^-} x^2\left[\frac{1}{x^2}\right] = 1$$

Discont. at x = 1
similarly for x = –1

$$f(x) = x^2\left(\frac{1}{x^2}-\left\{\frac{1}{x^2}\right\}\right) = 1 - x^2\left\{\frac{1}{x^2}\right\}$$

$$f(0^+) = \lim_{x\to 0^+} 1 - x^2\left\{\frac{1}{x^2}\right\} = 1$$

$f(0^{-1}) = 1$ But $f(0) = 0$

So discontinuous at $x = 0$

at $x = 2$, RHL = LHL = $f(2) = 0$

continuous at $x = 2$

Q.10 **(4)**

$$f(x) = \frac{|x-3|}{|x-2|} + \frac{1}{1+[x]}$$

$x \neq 2$ $1 + [x] = 0$

$[x] \neq -1,$ $x \in [1, 0)$

And [x] will be disoint. at every integer

So $x \in R - \{(-1, 0) \cup n, n \in I\}$

Q.11 **(2)**

f(x) should be a constant function.

Q.12 **(1)**

$$\text{RHL} = \underset{x \to 0^+}{\text{Lim}} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = 1\text{(Rationlize)}$$

$$\text{LHL} = \frac{1}{\sqrt{2}} f(g(x))$$

$$= \underset{x \to 0^-}{\text{Lim}} \frac{1}{\sqrt{2}} \frac{|\sqrt{2}\cos x| - |\sqrt{2}\sin x|}{\cos 2x}$$

$$= \underset{x \to 0^-}{\text{Lim}} \frac{1}{\cos x - \sin x} = 1$$

cont. at $x = 0$

Q.13 (3)

$$f\left(\frac{\pi^+}{4}\right) = \pi\left(\frac{\pi^+}{4}\right) + 1\pi \times 0 + 1 = 1$$

$$f\left(\frac{\pi}{4^-}\right) = f\left(\frac{\pi}{4}\right) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$$

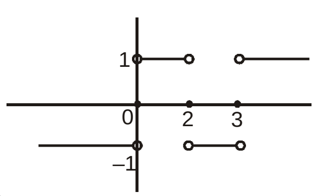
So jump $= 1 - \frac{\pi}{4}$

Q.14 **(3)**

$f(x) = \text{sgn}(x)$, $g(x) = x(x^2 - 5x + 6)$

$f(g(x)) = \text{sgn}(x(x^2 - 5x + 6)) = \text{sgn}(x(x-2)(x-3))$

$$g(g(x)) = \begin{cases} 1 \ ; & x(x-2)(x-3) > 0 \\ & x \in (0,2) \cup (3,\infty) \\ 0 \ ; & x(x-2)(x-3) = 0 \\ & x = 0,2,3 \\ -1 \ ; & x(x-2)(x-3) < 0 \\ & x \in (-\infty,0) \cup (2,3) \end{cases}$$



so, f(g(x)) is disc. at exactly points 0, 2 & 3

Q.15 (2)

$g(x) = x - [x]$ $f(0) = f(1)$

$h(x) = f(g(x))$

Let $x = a \in I$

$$h(a^+) = \lim_{x \to a^+} f(\{x\}) = f(0)$$

$$h(a^-) = \lim_{x \to a^-} f(g(x)) = f(1)$$

$h(a^+) = h(a^-)$ hence h(x) is continuous

Q.16 **(4)**

$$\text{RHL} = \underset{h \to 0}{\text{Lim}} \sin[\ell nh] = [-1, 1]$$

$$\text{LHL} = \underset{h \to 0}{\text{Lim}} \sin[\ell n\, h] = [-1, 1]$$

So DNE

Q.17 **(3)**

$f(x) = \text{Sgn}(4 - 2\sin^2 x - 2\sin x)$

$= \text{Sgn}[(\sin x + 2)(2 - 2\sin x)]$

$f(x) = 0$ when $x > \frac{\pi}{2}$

$= 1$ $x < \frac{\pi}{2}$

$= -1$ $\sin x > 1$ not possible

SO isolated point dicontinuity

Q.18 **(1)**

$g(x) = \tan^{-1}|x| - \cot^{-1}|x|$

$$f(x) = \frac{[x]}{[x+1]} \{x\}$$

$h(x) = |g(f(x))|$

$$\underset{x \to 0^-}{\lim(x)} |gf(0^-)| = \frac{n}{2}$$

$$\underset{x \to 0^+}{\lim(x)} = |gf(0^+)| = \frac{n}{2}$$

h is continuous at $x = 0$

Q.19 **(2)**

$$\lim_{x \to \infty} \frac{x^n - \sin x^n}{x^n + \sin x^n}$$

if $0 < x < 1$

then $x^n \to 0$ when $n \to \infty$

$f(x) = -1$

if $x > 1$

$$\lim_{x \to \infty} \frac{x^n - \sin x^n}{x^n + \sin x^n} = \frac{x^n + \sin x^n - 2\sin x^n}{x^n + \sin x^n}$$

$$= 1 - \frac{2\sin x^n}{x^n + \sin x^n}$$

Limit if x > 1

$\lim_{n\to\infty} = 1$

so has a finite discontinuity at x = 1

Q.20 **(1)**

RHL = $\lim_{x\to 0^+} \dfrac{[\{|x|\}]e^{x^2}\{[x+\{x\}]\}}{(e^{1/x^2}-1)Sgn(\sin x)}$

fraction part of greatest integer in always zero.
So RHL = LHL = 0
So coint. at x = 0

Q.21 **(4)**

RHL = $\lim_{x\to 0^+} \dfrac{\ln(e^{x^2}+2\sqrt{x}+1-1)(e^{x^2}+2\sqrt{x}-1)}{(e^{x^2}+2\sqrt{x}-1)\sqrt{x}}$

= 2

LHL = $\lim_{x\to 0^-} \dfrac{x[x]^2\,\ell n2}{\ell n(x+1)} = \ell n2$

Non-Removable disont at x = 0

Q.22 **(4)**

$$\lim_{x\to 0} \frac{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-....\right)-x\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+....\right)}{x^2}$$

= 0
f(x) is cont at x = 0

Q.23 **(2)**

f(x) = $x\left(\sqrt{x}-\sqrt{x+1}\right)$

f(0⁺) = $\lim_{h\to 0} \dfrac{h\left(\sqrt{h}-\sqrt{h+1}\right)}{h}$

= $\lim_{h\to 0} \dfrac{h-h-1}{\sqrt{h}+\sqrt{h+1}} = -1$

Q.24 **(2)**

$$f(x) = \begin{cases} x\dfrac{(3e^{1/x}+4)}{2-e^{1/x}} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

f '(0⁺) = $\lim_{h\to 0} \dfrac{h\dfrac{(3e^{1/h}+4)}{2-e^{1/h}}-0}{h}$

= $\lim_{h\to 0} \dfrac{3\left(1+\frac{1}{h}\right)+4}{2-1-\frac{1}{h}}$

= $\lim_{h\to 0} \dfrac{7h+3}{h-1} = 3$

f '(0⁻) = $\lim_{h\to 0} \dfrac{-h\dfrac{(3e^{-1/h}+4)}{2-e^{-1/h}}-0}{-h}$

= $\lim_{h\to 0} \dfrac{3\left(1-\frac{1}{h}\right)+4}{2-\left(1-\frac{1}{h}\right)}$

= $\dfrac{7h-3}{h+1} = -3$

so, not diff. at x = 0

Q.25 **(2)**

f(x) = $\dfrac{x}{\sqrt{x+1}-\sqrt{x}} = \dfrac{x(\sqrt{x+1}+\sqrt{x})}{x+1-x}$

f(x) = $x\left(\sqrt{x+1}+\sqrt{x}\right)$

Now, RHD

f(0⁺) = $\lim_{h\to 0} = \dfrac{f(0+h)-f(0)}{h}$

= $\lim_{h\to 0} \dfrac{h(\sqrt{h+1}-\sqrt{h})-0}{h}$

= 1

since ve⁻ values are not in domain of f(x) hence differentiability calculated by RHD Since RHD is finit hence f(x) is differentiable

Q.26 (2)

LHL

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(-n) = \lim_{h\to 0} \sin^{-1}[\cos h] = \dfrac{\pi}{2}$

RHL

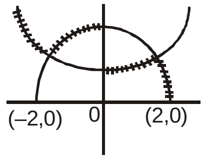
$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(h) = \lim_{h\to 0} \sin^{-1}[\cos h] = \dfrac{\pi}{2}$

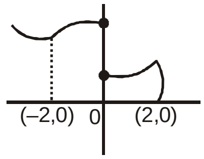
LHL = RHL
hence function is cont.

Q.27 **(4)**

$$f(x) = \begin{cases} \max(\sqrt{4-x^2}, \sqrt{1+x^2}) & -2 \le x \le 0 \\ \min(\sqrt{4-x^2}\ \sqrt{1+x^2}) & 0 < x \le 2 \end{cases}$$

y = 4 – x^2 , y = 1 + x^2





Q.28 **(3)**

f(x) =x – x^2

g(x) = $\begin{cases} \max f(t), & 0 \le t \le x,\ 0 \le x \le 1 \\ \sin \pi x, & x > 1 \end{cases}$

max f(t) will be obtained when t = x. so max (f(t)) = x – x^2

so $f'(1^+) = \lim_{h\to 0} \dfrac{\sin \pi(1+h) - 0}{h}$

$= \lim_{h\to 0} -\dfrac{\sin \pi h}{\pi h}\pi = -\pi$

$f'(1^-) = \lim_{h\to 0} \dfrac{(1-h)^2 - (1-h)^2 - 0}{-h}$

L.H.D. ≠ R.H.D. but both are finite so function is continuous, but not differentiable

Q.29 (4)

$f(0) = \lim_{x\to 0} f(x) = 0 - 1 + 0.\sin(-1) = -1$

$f(0^+) = \lim_{x\to 0} f(x) = 0 + 0 + 0.\sin 0 = 0 = f(0)$

f(x) is not continuous at x = 0

at x = 2,

$f(2^+) = 2 + 2 + 2 \sin 2 = 4 + 2 \sin 2$

$f(2^-) = 2 + 1 + 2 \sin 1 = 3 + 2 \sin 1$

f(x) is not continuous at x = 2

Q.30 **(2)**

$$f(x) = \begin{cases} \sqrt{x}\left(1+\sin\dfrac{1}{x}\right), & x > 0 \\ -\sqrt{x}\left(1+\sin\dfrac{1}{x}\right), & x < 0 \\ 0, & x = 0 \end{cases}$$

$f'(0^+) = \dfrac{f(0+h) - f(0)}{h} \Rightarrow \lim_{h\to 0}$ d

$= \lim_{h\to 0} \dfrac{\left(1+\sin\frac{1}{h}\right)}{\sqrt{h}}$ = not defined ⇒ not differentiable

$f(0^+) = \lim_{h\to 0} \sqrt{h}\left(1+\sin\dfrac{1}{h}\right)$

$= \lim_{h\to 0} \dfrac{h}{\sqrt{h}}\left(1+\sin\dfrac{1}{h}\right)$

$= \lim_{h\to 0} \left(\dfrac{1}{\sqrt{h}}\right)\dfrac{\left(1+\sin\frac{1}{h}\right)}{(1/h)}$

$= \lim_{h\to 0} \dfrac{1}{\sqrt{h}}\left\{h + h\sin\dfrac{1}{h}\right\}$

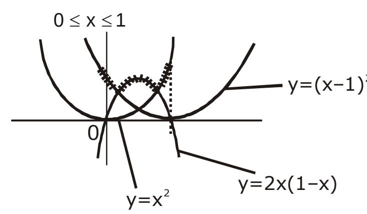
= 0

$f(0^-) = \lim_{h\to 0} -\sqrt{h}\left(1-\sin\dfrac{1}{h}\right)$

= 0

Q.31 **(3)**

f(x) = max {x^2, $(x - 1)^2$, 2x(1 – x)}



so, (3)

Q.32 **(3)**

f(x) = $x^3 - x^2 + x + 1$

g(x) = $\begin{cases} \max (f(t)); 0 \le t \le x \text{ for } 0 \le x \le 1 \\ x^2 - x + 3; 1 < x \le 2 \end{cases}$

max {f(t)} will be obtained when ‘t’ would be max.

so, t = x.

so, max {f(t)} = $x^3 - x^2 + x + 1$

$f'(1^+) = \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \dfrac{(1+h)^2 - (1+h) + 3 - 2}{h}$

= not defined

so not derivable

Now check cont by,

$f(1^+) = \lim_{h\to 0} f(1 + h)$

$= \lim_{h\to 0} (1 + h)^2 - (1 + h) + 3$

= 3

& f(1) = 2

$f(1^+) \ne f(1)$

so f(x) is not continuous

Q.33 **(2)**

differentiability at x = 0

$$f'(0^+) = \lim_{h\to 0} \frac{\log_a \left(a\left|[h]+[-h]\right|\right)^h \left(\frac{a^{\left(\frac{2}{\left(\frac{[h]+[-h]}{h}\right)}-5\right)}}{3+a^{1/h}} \right) - 0}{h}$$

$$= \lim_{h\to 0} \frac{h(\log_a a)\left(\frac{a^{\frac{2h}{-1}-5}}{3+a^{1/h}}\right)}{h} = \lim_{R\to 0} \frac{a^{-(2h+5)}}{3+a^{1/h}} = 0$$

$$f'(0^-) = \lim_{h\to 0} \frac{\log_a \left(a\left|[-h]+[h]\right|\right)^{(-h)} \left(\frac{a^{\left(\frac{2}{\left(\frac{[-h]+[h]}{+h}\right)}-5\right)}}{3a^{1/h}} \right) - 0}{h}$$

$$= \lim_{h\to 0} \frac{(-h)\cdot a^{-(2h+5)}}{h\cdot\left(3+a^{1/h}\right)} = 0$$

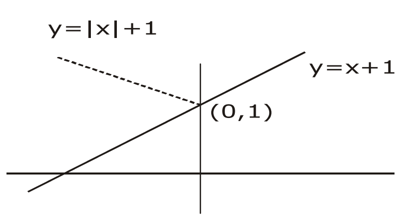
So continious & differentiable.

Q.34 **(2)**

$f'(2^-) = f'(2^+) = 2$

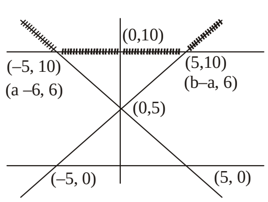
& $f'(3^+) = f'(3^-) = \frac{21}{4}$

Q.35 **(3)**



Q.36 **(1)**

Q.37 **(2)**



$0 < a < b$
$a = 5$
$b = 10$
$f(x) = \max(5 - x, 5 + x, 10)$

$a - x = b$
$a + x = b$
f(x) is not diff at $x = a - b$ & $x = b - a$

Q.38 **(4)**

$f'(O^+) = p + q$ ....(1)
$f'(O^-) = -p + q$ ....(2)
$f'(O^+) = f'(O^-) \Rightarrow p + q = 0, r \in R$

Q.39 **(1)**

$g(x) = [x] + 1$
$h(x) = g(\sin x) = [\sin x] + 1$

[sinx] is discont at $x = \frac{\pi}{2}$

$\Rightarrow$ [sinx] + 1 is also a discont at $x = \frac{\pi}{2}$

Q.40 **(2)**

$f(x) = [\tan^2 x]$

RHL = $\lim_{x\to O^+} [\tan^2 x] = 0$

LHL = $\lim_{x\to O^-} [\tan^2 x] = 0$: $f(x) = 0$

So cont. at x = 0

Q.41 **(3)**

$f(x) = [x]^2 + \sqrt{(x-[x])^2}$

DIscont. at every integer because [x] is discont. at every integer.
But f(x) is cont. at x =1
So option (3) is correct.

Q.42 **(2)**

$f'(0^+) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \ell \neq 0$

$f'(0^-) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = -\ell \neq 0$

Non. diff. But cont. at x = 0

Q.43 **(3)**

RHD = $\lim_{h\to 0} \frac{\frac{h}{\sinh}-1}{h} = \lim_{h\to 0} \frac{h-\sinh}{h^2} = 0$

LHD = $\lim_{h\to O} \frac{\frac{-h}{\sinh}-1}{h} = \lim_{h\to 0} \frac{-h-\sinh}{h^2} \to \infty$

Non. diff. at x = 0
RHL = 1 Discont.
LHL = –1

Q.44 **(1)**

LHD (x = 1) = RHD (x = 1)
1 = 2a + b ....(1)
LHL (x = 1) RHL (x = 1)

1 = a + b + c ....(2)
b = 1-2a, c = a

Q.45 **(4)**
RHD (at x = 0) = 0; LHD = 1
RHD (at x = 1) = 2; LHD = 2
RHL (at x = 0) = 0 = LHL
RHL (at x = 1) = LHL (x = 1)
Diff. and cot. at x = 1
Non diff. x = 0 but cont. at x = 0

Q.46 **(2)**
If f is differentiable everywhere.
then |f| will also be diff. everywhere.
and if two fns. are diff. then sum of then will also be diff. everywhere

Q.47 **(4)**
f(x + y) = f(x) . f(y), f(3) = 3
f '(0) = 11, f(3) = ?

$$f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x)\cdot f(h)-f(x)}{h}$$

$$= f(x) \,.\, \lim_{h\to 0} \frac{f(h)-1}{(h)}$$

$$f'(3) = f(3) \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

f '(3) = f(3) . f '(0)
f '(3) = 3 × 11 = 33
[∵ f(0) = f(0) . f(0) ⇒ f(0) = 1]

Q.48 **(4)**
f(x + 2y) = f(x) + f(2y) + 2xy

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x)+f(h)-f(x)+2xy}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(0+h)-(0)}{h} + 2x$$

f′(x) = f '(0) + 2

Q.49 **(3)**

Put y = 0 ⇒ $f\left(\frac{x}{3}\right) = \frac{f(x)}{3}$ ⇒ f(3x) = 3f(x)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0}\frac{\frac{f(3x)+f(3h)}{3}-f(x)}{h} \quad \lim_{h\to 0}\frac{\frac{8f(x)+3f(x)}{3}-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(h)-f(0)}{h} = f'(0)$$

f'(x) = 3 ⇒ f(x) = 3x + c ⇒ f(x) = 3x

Q.50 **(3)**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x)+f(h)+xh-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(h)}{h} + x$$

f'(x) = x + 3

$f(x) = \frac{x^2}{2} + 3x + c$ f(0) = 0

$f(x) = \frac{x^2}{2} + 3x$ ⇒ c = 0

Q.51 **(4)**

Put x = 0, y =0 ⇒ $f(0) = \frac{4}{7}$

Now put y = 0

$$f\left(\frac{x}{3}\right) = \frac{4-2[f(x)+f(0)]}{3}$$

⇒ 3f(x) = 4 – 2 [f(3x) + 6(0)]

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Now proceed as in question (28)

$f(x) = \frac{4}{7}$

Q.52 **(2)**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x).f(h)-f(x)}{h} = f(x) \lim_{h\to 0} \frac{f(h)-f(0)}{h}$$

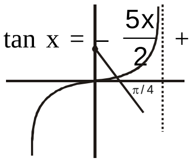
f'(x) = f(x) . f' (0)
f'(x) = 2f(x)
ln f(x) = 2x + C : C = 0
$f(x) = e^{2x}$

Q.53 **(4)**
f(x + y) = f(x).f(y)
differentiate w.r.t. x
f′(x + y) = f′(x) . f(y)
put x = 0, y = 5
f′(5) = f′(0) . f(5)
= 3 . 2
∴ f′(5) = 6

Q.54 **(2)**

2 tan x + 5x – 2 = 0

$\tan x = -\frac{5x}{2} + 1$



Q.55 **(3)**

By using L' Hospital rule

$= \lim\limits_{x\to 0} \frac{2f'(x) - f'(2x) + 4f'(4x)}{2x}$

Again

$= \lim\limits_{x\to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2} = 12$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 **(C)**

$f\left(\frac{\pi}{2}\right) = \lim\limits_{h\to 0} \frac{1-\cos h}{4h^2} \frac{\ell n(\cos h)}{\ell n[1+4h^2]}$

$= \lim\limits_{h\to 0} \frac{2}{16\times 16}\left(\frac{\sin^2 h/2}{h^2/2}\right) . \frac{4h^2}{\ell n(1+4h^2)} .$

$\frac{\ell n(1-2\sin^2 h/2)}{2\sin^2 h/2} . \frac{2\sin^2 h/2}{h^2/2}$

$= \frac{1}{64} . 1. 1.(-1) .1 = -\frac{1}{64}$

Q.2 **(C)**

By using rationlization

$f(0) = \lim\limits_{x\to 0} \frac{(a^2 - ax + x^2) - (a^2 + ax + x^2)}{(a+x) - (a-x)}$

$\times \frac{(\sqrt{a+x} + \sqrt{a-x})}{(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2})}$

$= -\frac{-2ax}{2x}\left(\frac{2\sqrt{a}}{2a}\right)$

$f(0) = -\sqrt{a}$

Q.3 **(A)**

g(x) = x – [x] = {x} ∈ [0, 1)

g(x) is discontinuous only at x ∈ I

Now h(x) = fog(x)

h(x) is continuous ∀ x ∈ R – I

Let x ∈ I, consider x = n

h(n) = f [g(n)] = f(0)

$\lim\limits_{x\to n^-} h(x) = \lim\limits_{x\to n^-} f(\{x\}) = f(1) = f(0)$

$\lim\limits_{x\to n^+} h(x) = \lim\limits_{x\to n^+} f(\{x\}) = f(0)$

⇒ h(x) is continuous ∀ x ∈ I

⇒ h(x) is continuous ∀ x ∈ R

Q.4 (D)

Q.5 (C)

Q.6 **(C)**

$\text{RHL} = \lim\limits_{x\to \frac{\pi^+}{2}} f(x) = \lim\limits_{h\to 0} \frac{\sin\left\{\cos\left(\frac{\pi}{2}+h\right)\right\}}{h}$

$= \lim\limits_{h\to 0} \frac{\sin\{-\sin b\}}{h}$

$= \lim\limits_{h\to 0} \frac{\sin(1-\sin h)}{h} \to \infty$

DNE

Q.7 **(B)**

Q.8 **(C)**

$\text{RHL} = \lim\limits_{h\to 0} \frac{a^h - 1}{h} = \ell na$

$\text{LHL} = \lim\limits_{h\to 0} \frac{a^{-1-h} - 1}{-1-h} = 1 - \frac{1}{a}$

f(x) = ℓna ⇒ Discont. at x = 0

Q.9 **(C)**

x = tanθ

$f(x) = \sin^{-1}(\text{cosec } 2\theta)$

$\sin^{-1}$ is defined for [–1, 1] only

So cosec 2θ = 1 & –1 only

Hence neither continious Nor differentiable at x = 1

Q.10 (A)

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1} x^2 e^{2(x-1)} = 1$

f(1) = 1

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1}$ a sgn (x +1) cos2(x–1) + bx^2

= a.1.1+ b

for continuity a + b = 1

LHD (x = 1) is $\lim\limits_{h\to 0} \frac{(1-h)^2 e^{-2h} - 1}{h}$

$= \lim\limits_{h\to 0} 2e^{-2h} + he^{-2h} + \left(\frac{e^{-2h} - 1}{h}\right)$

= 2 + 0 + 2 = 4

RHD (x = 1)

is $\lim\limits_{h\to 0} \frac{a\,\text{sgn}(2+h)\cos 2h + b(1+h)^2 - 1}{h}$

$= \lim_{h\to 0} \frac{a\cos 2h + b + bh^2 + 2bh - (a+b)}{h}$ =

$\lim_{h\to 0} a\left(\frac{\cos 2h - 1}{h}\right) + bh + 2b$ = 2b

f(x) is differentiable at x = 1 if 2b = 4

$\Rightarrow$ b = 2 a = –1

Q.11 **(B)**

f(x) = [x] [sin πx], x ∈ (–1, 1)

$= \begin{cases} 1 & , \quad x \in (-1, 0) \\ 0 & , \quad x \in [0, 1) \end{cases}$

f(x) is continuous in (–1, 0)

Q.12 **(D)**

$g\left(\frac{1}{2}\right) = f(1) = 0$

$f\left(\frac{1^+}{2}\right) = f[1^+] = f(1) = 0$

$g\left(\frac{1^-}{2}\right) = f[0] = f(0) = 1$

Discont. at x = $\frac{1}{2}$

Q.13 **(C)**

$\frac{|f(x) - f(y)|}{x - y} \le (x - y)$

$\lim_{x\to y} \frac{|f(x) - f(y)|}{x - y} \le \lim_{x\to y} (x - y)$

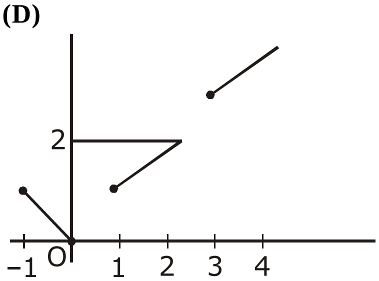
f'(x) = 0

$\Rightarrow$ f(x) = C

$\Rightarrow$ f(0) = 0

$\Rightarrow$ C = 0 f(1) = 0

Q.14 **(D)**



Discont at x = 1, 2, 3

Non. diff. at x = 1, 2, 3

Q.15 **(D)**

[n + p sin x] =n [p sin x]

$\therefore$ [p sin x] is non. diff. where p sin x is as integer but P is prime and 0 < sin x ≤ 1 [0 < x < π]

$\therefore$ p sin x is an integer only when

$\sin x = \frac{r}{p}$; where 0 < r ≤ p and r ∈ N

For r = p ; sin x = 1 $\Rightarrow$ x = $\frac{\pi}{2}$ in (0, π)

For 0 < r < p ; sin x = $\frac{r}{p}$

$x = \sin^{-1}\left(\frac{r}{p}\right)$ or $\pi - \sin^{-1}\left(\frac{r}{p}\right)$

Number of such values of x = P – 1 + P – 1 = 2P – 2

Total No. of points = 2P – 2 + 1 = 2P – 1

Q.16 **(D)**

$f(x) = [\sin[x]] = \begin{cases} [\sin 0] = 0 & 0 \le x < 1 \\ [\sin 1] = 0 & 1 \le x < 2 \\ [\sin 2] = 0 & 2 \le x < 3 \\ [\sin 3] = 0 & 3 \le x < 4 \\ [\sin 4] = -1 & 4 \le x < 5 \\ [\sin 5] = -1 & 5 \le x < 6 \\ [\sin 6] = -1 & 6 \le x < 2\pi \end{cases}$

f(x) is discontinuous at (4, –1)

JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 **(C, D)**

Q.2 **(A, B, C)**

f(x) = [x] and $g(x) = \begin{cases} 0 & , \quad x \in I \\ x^2 & , \quad x \in R - I \end{cases}$

$\lim_{x\to 1} g(x) = \lim_{x\to 1} x^2 = 1$, but g(1) = 0

$\lim_{x\to 1} f(x) = \lim_{x\to 1} [x]$ does not exist since LHL = 0 and RHL = 1

gof(x) = g([x]) = 0

$\Rightarrow$ gof(x) is continous for all values of x

$fog = \begin{cases} 0 & , \quad x \in I \\ [x^2] & , \quad x \in R - I \end{cases}$

fog(1) = 0 , $\lim_{x\to 1-} fog(x) = 0$, $\lim_{x\to 1^+} fog(x) = 1$

fog is not continous at x = 1

Q.3 **(A, B, C)**

$\lim_{x\to1^+} |x-3| = 2$

$\lim_{x\to1^-} \left(\frac{x^2}{4}\right) - \left(\frac{3x}{2}\right) + \frac{13}{4} = 2$

function is cont. at x = 1
function is also diff. at x = 1 and will be cont. at x = 3

Q.4 **(A, B, C)**

$f(x) = [x] + \sqrt{x-[x]}$

$f(x) = [x] + \sqrt{\{x\}} \Rightarrow x - \{x\} + |\{x\}| \to$ always positive
f(x) = x
f(x) is continuous on R, R^+, R – I

Q.5 **(B, D)**

$f(x) = \lim_{n\to\infty} (\sin x)^{2n}$

$$f(x) = \begin{cases} 1 \ ; & x = \pi/2 \\ 0 \ ; & x < \pi/2 \\ \infty \ ; & x > \pi/2 \end{cases}$$

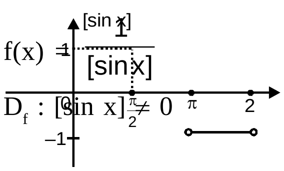
$$\left.\begin{matrix} f\left(\frac{\pi}{2}^+\right) = \infty \\ f\left(\frac{\pi}{2}^-\right) = 0 \end{matrix}\right\} \text{function is not cont. at } x = \frac{\pi}{2}$$

function is discont at $x = \frac{\pi}{2}$ & infinit number of points.

Q.6 **(A, B, D)**

$f(x) = \frac{1}{[\sin x]}$

$D_f : [\sin x] \neq 0$



$x \in (2n\pi + \pi, 2n\pi + 2\pi) \cup \left\{2n\pi + \frac{\pi}{2}\right\}$

cont. when $x \in (2n\pi + \pi, 2n\pi + 2\pi)$
f(x) has the period of 2π

Q.7 (B, C, D)
(A) f(x) is continuous no where
(B) g(x) is continuous at x = 1/2
(C) h(x) is continuous at x = 0
(D) k(x) is continuous at x = 0

Q.8 **(A, B, C)**

Q.9 **(B, D)**

$$f(x) = \begin{cases} 3 - \left[\cot^{-1}\left(\frac{2x^3-3}{x^2}\right)\right] & x > 0 \\ \{x^2\}\cos(e^{1/x}) \quad \text{for} & x < 0 \end{cases}$$

RHL $\lim_{x\to0^+} 3 - \left[\cot^{-1}\frac{2x^2-3}{x^2}\right] = 3 - 3 = 0$

$\cot^{-1}(-\infty) \to [\pi] = 3$

LHL $\lim_{x\to0^-} \{x^2\} \cos e^{1/x}$

x = 0 – h

$\lim_{h\to0} \underset{0}{\{h^2\}} \times \underbrace{\cos e^{-1/h}}_{1} = 0$

Q.10 **(B, C)**

$f\left(\frac{1}{4^n}\right) = (\sin e^n)\, e^{-n^2} + \frac{n^2}{1+n^2}$

put n = ∞
f(0) = [{a finite quantity b/w (–1, 1)} × 0] + 1 = 1

Q.11 **(C, D)**

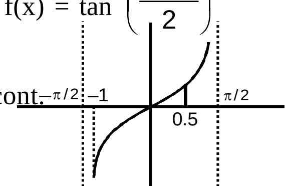
$f(x) = \frac{x}{2} - 1$ on $[0, \pi]$

$f(x) = \frac{x-2}{2} : \frac{1}{f(x)} = \frac{2}{x-2}, \ 0 \le \frac{x}{2} \le \frac{\pi}{2}$

$f^{-1}(x) = 2(1+x)$ is continuous $-1 \le \frac{x}{2} - 1 < \frac{\pi}{2} - 1 \approx 0.5$

$\tan f(x) = \tan\left(\frac{x-2}{2}\right)$

is cont.



$\frac{1}{f(x)}$ will discont at x = 2

Q.12 **(A, B, C)**

Q.13 (A, B, C)

Q.14 **(B, D)**

$$y = f(x) = \begin{cases} (\sin^{-1}x)^2 \cos 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

f(x) can be discontinuous only at x = 0 in [–1,1]
So we check only at x = 0

LHD (x = 0) = $\lim\limits_{h\to 0}\dfrac{(\sin^{-1}-h)^2\cos\left(-\frac{1}{n}\right)-0}{-h}$

$\lim\limits_{h\to 0}-\left(\dfrac{\sin^{-1}-h}{n}\right)^2 . h\cos\left(\dfrac{1}{n}\right)$ = –1. 0. [finite quantity between [–1,1]] = 0

RHD (x = 0) is $\lim\limits_{h\to 0} h\dfrac{(\sin^{-1}-h)^2}{n^2}.\cos\left(\dfrac{1}{n}\right)=0$

Hence f(x) is differentiable as well as continuous in [–1,1]

Q.15 **(A, C)**

f(x) = $\sum\limits_{k=0}^{n} a_k\,|x|^k$

f(x) = $a_0|x|^0 + a_1|x| + a_2|x|^2 + ... + a_n|x|^n = f(|x|)$
f(x) is cont. at x = 0 ∀ all is
2k + 1 means all odd a_i's
f(x) = $a_0 + a_2x^2 + a_4x^4 + ...$
f(x) will be diff. at x = 0

Q.16 **(A, B, D)**
f(0) = 0
$f(0^+) = [0^+] = 0$
$f(0^-) = [0^+] = 0$
$f(0^-) = [0^+] = 0$
So f(x) is cont. at x = 0
f(1) =0
$f(1^+)$ = –1 So discont. at x = 1
⇒ Non. diff. at x = 1

Q.17 **(A, B, D)**

f(x) = $\sqrt{1-\sqrt{1-x^2}}$

$D_f : 1 - x^2 \ge 0 \Rightarrow -1 \le x \le 1$
RHL (at x = 0) = 0
LHL (at x = 0) = 0 cont. at x = 0

RHD = $\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$= \lim\limits_{h\to 0}\dfrac{\sqrt{1-\sqrt{1-h^2}}-0}{h}$

$= \lim\limits_{h\to 0}\dfrac{1}{h}\sqrt{1-\sqrt{1-h^2}\times\dfrac{1+\sqrt{1-h^2}}{1+\sqrt{1+h^2}}}$

$= \lim\limits_{h\to 0}\dfrac{1}{h}\sqrt{\dfrac{1-1+h^2}{1+\sqrt{1+h^2}}} = \dfrac{1}{2}$

LHD = $-\dfrac{1}{2}$

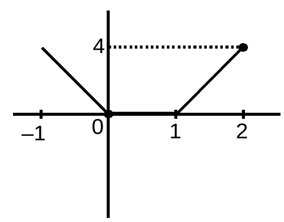
Q.18 **(A, C)**

f(x) = $\lim\limits_{n\to\infty}\dfrac{1-x^n}{1+x^n} = \begin{cases}1 \;;\; x<1\\ \infty \;;\; x>1\\ 0 \;;\; x=1\end{cases}$

$f(1^+) = \infty$
$f(1^-) = 1$
f(x) is a constant in 0 < x < 1
$f'(0^+) \ne f'(0^-)$ not diff. at x = 1

Q.19 **(A,C)**
f(x) = |[x]x| –1 ≤ x ≤ 2

f(x) = $\begin{cases}-x & -1\le x<0\\ 0 & 0\le x<1\\ x & 1\le x<2\\ 4 & x=2\end{cases}$ | $\lim\limits_{x\to 0^+} 0 = 0 : \lim\limits_{x\to 0^-} 0 = 0$
cont. at x = 0
Not diff. at x = 2



Q.20 **(A, B)**

f(x) = 1 + x . [cos x] $0 < x \le \dfrac{\pi}{2}$

$f\left(\dfrac{\pi}{2}\right) = 1 = f\left(\dfrac{\pi}{2}^-\right)$

function is cont. is $\left(0, \dfrac{\pi}{2}\right]$

$f'\left(\dfrac{\pi}{2}^-\right) = \lim\limits_{h\to 0}\dfrac{1-h[\cos(-h)]-1}{-h} = 1$

diff. at x = $\dfrac{\pi}{2}$

Q.21 **(B, D)**

f(x) = $(\sin^{-1} x)^2 . \cos\left(\dfrac{1}{x}\right)$ if x ≠ 0
= 0 if x = 0

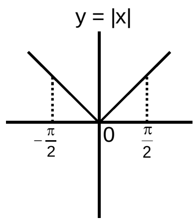
LHL = RHL = $\lim\limits_{x\to 0}(\sin^{-1} x)^2 \cos\left(\dfrac{1}{x}\right)$
= 0 × [a finite quantity b/w (–1, 1)]
= 0

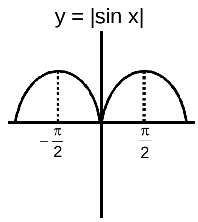
$$f'(0^+) = \lim_{h\to 0} \frac{(\sin^{-1}h)^2 \cos\left(\frac{1}{h}\right)}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sin^{-1}h}{h}\right) (\sin^{-1} h) \cos(1/h)$$

= 1 × (0) × (a finite quantity)
= 0
$f'(0^-) = 0$

Q.22 **(B, D)**





Not diff. at x = 0

Q.23 **(A, B, C)**
$f(x) = 3(2x + 3)^{2/3} + 2x + 3$

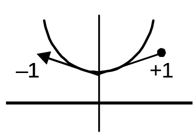
$f\left(\frac{-3}{2}\right) = 0 - 3 + 3 = 0$

cont. every where
$f'(x) = -2(2x + 3)^{-1/3} + 2$

$$= -\frac{2}{(2x+3)^{1/3}} + 2$$

at $x = -\frac{3}{2}$; $f'(x)$ is not defined

Q.24 **(B, D)**
$f(x) = 2 + |\sin^{-1} x|$



1 function is continuous everywhere in its domain
1 but f(x) is not diff. at x = 0

Q.25 **(A, B, D)**

$f(x) = x^2 \sin\left(\frac{1}{x}\right)$, $x \neq 0$

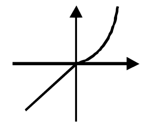
= 0 , x = 0
cont. at x = 0
$f(0^+) = f(0^-) = f(0) = 0$

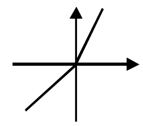
$$f'(0^+) = \lim_{h\to 0} \frac{h^2 \sin 1/h}{h} = \lim_{h\to 0} h \sin\frac{1}{h} = 0$$

$f'(0^-) = 0$
Diff. at x = 0

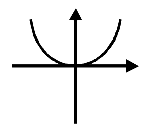
Q.26 **(A, B, D)**
$\sin^{-1} x + |y| = 2y$
$\sin^{-1} x = 2y - y$
$y = \sin^{-1} x$
y is defined for $-1 \le x \le 1$

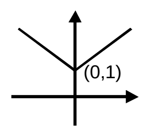
Q.27 **(A, B, D)**

(A) 

(B) 

(C) $h(x) = x^2 \quad x \ge 0 \qquad = -x^2 \; x < 0$



(D) 

Comprehension # 1

Q.28 **(C)**

Q.29 **(C)**

Q.30 **(C)**
(28 to 30)

$g(t) = \lim_{x\to 0} (1 + a\tan x)^{t/x}$

$g(t) = e^{\lim_{x\to 0} \frac{t}{x} a\tan x} = e^{\lim_{x\to 0} ta \frac{\tan x}{x}}$
$g(t) = e^{ta} = e^{ta}$
$g(x) = e^{ax}$
∵ a = 2, $g(x) = e^{2x}$
$g(2) = e^4$

$$f(x) = \begin{cases} x e^{ax} & , \; x \le 0 \\ x + ax^2 - x^3 & , \; x > 0 \end{cases}$$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x + ax^2 - x^3 = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x e^{ax} = 0$

f(0) = 0
$a \in (0, \infty)$

$$f'(0^+) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{h + ah^2 - h^3}{h} = 1$$

$$f'(0^-) = \lim_{h\to 0^-} \frac{f(o-h)-f(o)}{-h} = \lim_{h\to 0^-} \frac{he^{-ah}}{h} = 1$$

Comprehension # 2

Q.31 **(B)**

Q.32 **(C)**

Q.33 **(B)**

(31 to 33)

Given function f(x) can be rewritten as, f(x) =

$$\begin{cases} 0 & , \quad x < -1 \\ 1+x & , \quad -1 \le x \le 0 \\ 1-x & , \quad 0 < x \le 1 \\ 0 & , \quad x > 1 \end{cases}$$

$$\Rightarrow f(x-1) = \begin{cases} 0 & , \quad x-1 < -1 \\ 1+(x-1) & , \quad -1 \le x-1 \le 0 \\ 1-(x-1) & , \quad 0 < x-1 \le 1 \\ 0 & , \quad x-1 > 1 \end{cases} \text{ or } f(x$$

$$-1) = \begin{cases} 0 & , \quad x < 0 \\ x & , \quad 0 \le x \le 1 \\ 2-x & , \quad 1 < x \le 2 \\ 0 & , \quad x > 2 \end{cases}$$

$$\text{also, } f(x+1) = \begin{cases} 0 & , \quad x+1 < -1 \\ 1+(x+1) & , \quad -1 \le x+1 \le 0 \\ 1-(x+1) & , \quad 0 < x+1 \le 1 \\ 0 & , \quad x+1 > 1 \end{cases} \text{ or } f(x$$

$$+1) = \begin{cases} 0 & , \quad x < -2 \\ 2+x & , \quad -2 \le x \le -1 \\ -x & , \quad -1 < x \le 0 \\ 0 & , \quad x > 0 \end{cases}$$

Now, g(x) = f(x – 1) + f(x + 1) =

$$\begin{cases} 0 & , \quad x < -2 \\ 2+x & , \quad -2 < x \le -1 \\ -x & , \quad -1 < x \le 0 \\ x & , \quad 0 < x \le 1 \\ 2-x & , \quad 1 < x \le 2 \\ 0 & , \quad x > 2 \end{cases}$$

It is easy to check that g(x) is continuous for all x $\in$ R and non-differentiable at x = – 2, –1, 0, 1, 2.

Q.34 (A) → (p, r, s), (B) → (p, r, s), (C) → (q, r, s), (D) → (r, s)

Consider the graph of 2 cos x in (–π, π). 2 cos x is integer at 9 points.

[2 cos x] is discontinuous at 7 points in (–π, π)

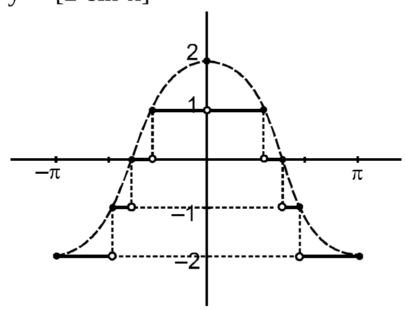
Similarly from graph of 2sinx, we can observe that [2 sin x] is discontinuous at 7 points (continuous at –π/2, π)

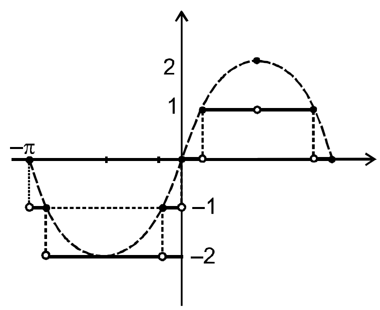
[2 tan x/2] is discontinuous at 4 points (continuous at –π/2)

[3 cosec x/3] is discontinuous at 4 points (continuous at π/2)

y = [2 cos x]

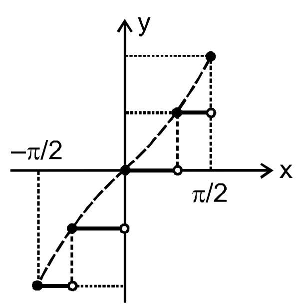
y = [2 sin x]

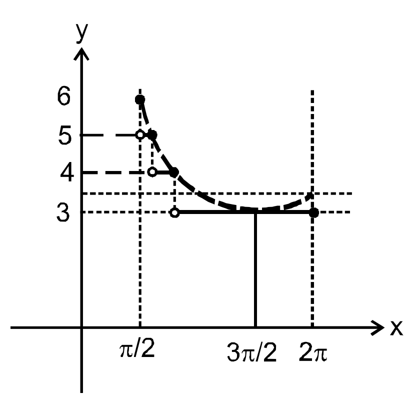




y = [2 tan x/2]

y = [3 cosec x/3]





Q.35 **(A)–S ; (B)–P,T ; (C)–P,R ; (D)–R,S**

(A) Total function = 3^4 = 81

onto function = $\frac{4!\times 3}{2!2!}$ = 36

Into function = 81 – 36 = 45

So difference = 45 – 36 = 9

(B) $\lim\limits_{x\to -2} f(x) = -1$ $f'(x) < 0$ always

So function is decreasing

So $\lim\limits_{x\to -2^+}\left\{\frac{f(x)}{x}\right\} = 1$ & $\lim\limits_{x\to -2^-}\left\{\frac{f(x)}{x}\right\} = 0$

(C) Function will be non differentiable at x = 1 & x = 3

(D) $\lim\limits_{x\to 0}\frac{f_3(x)}{2x}$

$$\frac{\tan\frac{x}{2}\,(1+\sec x)(1+\sec 2x)(1+\sec 4x)\,(1+\sec 8x)}{2x}$$

= 8 $2^k = 8$ k = 3

Q.36 (A) → (p, q, r), (B) → (p, r, s), (C) → (p, r, s), (D) → (p, r, s)

(A) $f(x) = |x^3|$ is continuous and differentiable

(B) $f(x) = \sqrt{|x|}$ is continuous

$$f'(x) = \frac{1}{2\sqrt{|x|}}\cdot\frac{x}{|x|}$$

{does not exist at x = 0}

(C) $f(x) = |\sin^{-1} x|$ is continuous

$$f'(x) = \frac{\sin^{-1}x}{|\sin^{-1}x|}\cdot\frac{1}{\sqrt{1-x^2}}$$

{does not exist at x = 0}

(D) $f(x) = \cos^{-1}|x|$ is continuous

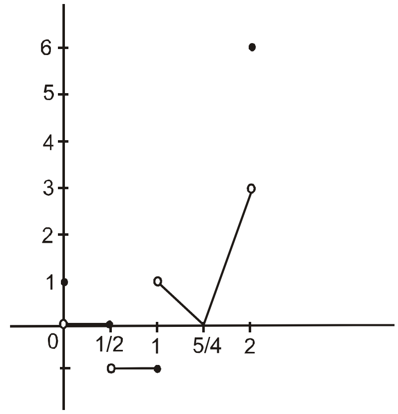
$$f'(x) = \frac{-1}{\sqrt{1-x^2}}\cdot\frac{x}{|x|}$$

{does not exist at x = 0}

NUMERICAL VALUE BASED

Q.1 [4]

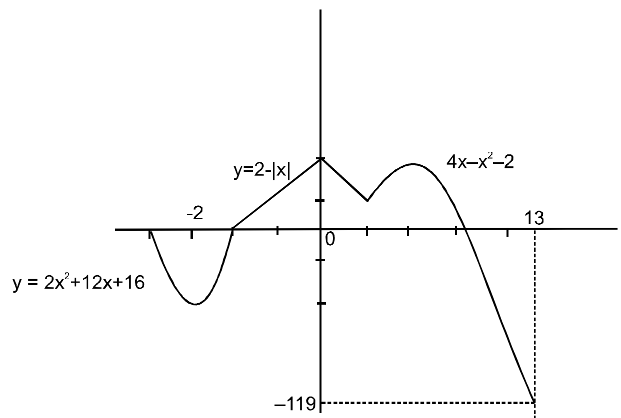
$$f(x) = \begin{cases} 1 & , \quad x = 0 \\ 0 & , \quad 0 < x \le 1/2 \\ -1 & , \quad 1/2 < x \le 1 \\ 5-4x & , \quad 1 < x < 5/4 \\ 4x-5 & , \quad 5/4 \le x < 2 \\ 6 & , \quad x = 2 \end{cases}$$



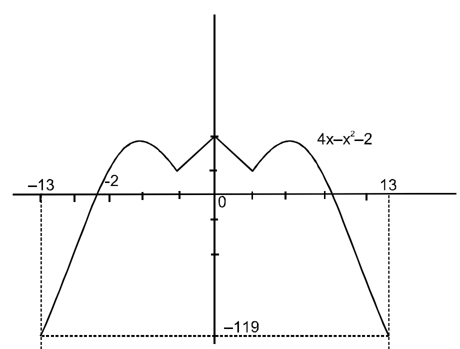
f(x) is discontinous at x = 0, 1/2, 1, 2 in [0, 2]

Q.2 [26]

y = f(x)



y = f(|x|)



Q.3 [36]

$$f\left(\frac{\pi}{2}\right) = \lim_{h\to 0}\frac{1-\cosh}{4h^2}\,\frac{\ell n(\cosh)}{\ell n[1+4h^2]}$$

$$= \lim_{h\to 0}\frac{2}{16\times 16}\left(\frac{\sin^2 h/2}{h^2/2}\right)\cdot\frac{4h^2}{\ell n(1+4h^2)}.$$

$$\frac{\ell n(1-2\sin^2 h/2)}{2\sin^2 h/2} \cdot \frac{2\sin^2 h/2}{h^2/2}$$

$$= \frac{1}{64} \,.\, 1.\ 1.(-1)\ .1 = -\frac{1}{64}$$

$\Rightarrow \quad \alpha^\beta = 64 = 2^6, 4^3, 8^2, 64^1$

Q.4 [16]

we have

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0^+} (\sin(-h) + \cos(-h))^{\text{cosec}(-h)}$$

$$= \lim_{h\to 0^+} (\cosh - \sinh)^{-\text{cosech}}$$

$$= \lim_{h\to 0^+} (1 + (\cosh - \sinh - 1))^{\frac{1}{(\cosh-\sinh-1)}\cdot\frac{(\cosh-\sinh-1)}{(-\sinh)}}$$

$$= \lim_{h\to 0^+} e^{\frac{\cosh-\sinh-1}{-\sinh}} = e$$

Now we have $\lim_{x\to 0^+} f(x) = \lim_{h\to 0^+} \dfrac{e^{\frac{1}{h}} + e^{2/h} + e^{3/h}}{ae^{-2+1/h} + be^{-1+3/h}}$

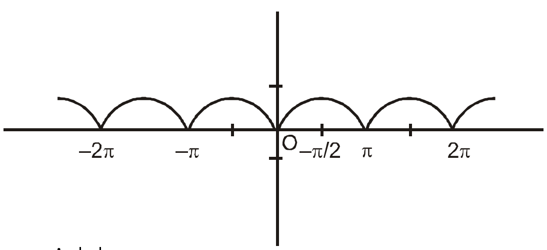
$$= \lim_{h\to 0^+} \frac{e^{-\frac{2}{h}} + e^{\frac{-1}{h}} + 1}{(ae^{-2})e^{-2/h} + (be^{-1})} = \frac{e}{b}$$

If 'f' is continuous at x = 0 , then

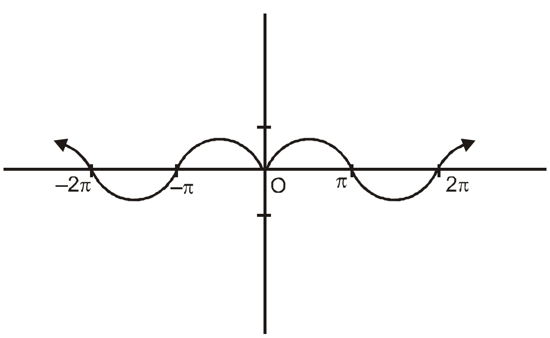
$e = a = \dfrac{e}{b}$ gives a = e and b = 1

Q.5 [7]

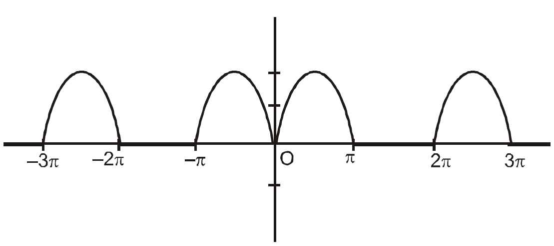
y = |sinx|



y = sin|x|



y = f(x) = |sinx| + sin|x|



f(x) is continous every where

f(x) is not differentiable at x = nπ

f(x) is not periodic

Q.6 [4]

Differentiability at x = 1

$$f'(1^-) = \lim_{h\to 0} \frac{\dfrac{\sin[(1-h)^2]\,\pi}{(1-h)^2 - 3(1-h) + 8} + a\,(1-h)^3 + b - (a+b)}{-h}$$

$$= \lim_{h\to 0} \frac{a(1-h)^3 - a}{-h} \left(\frac{0}{0}\text{ form}\right) = \lim_{h\to 0} \frac{3a\,(1-h)^2}{1}$$

$f'(1^-) = 3a$

$$f'(1^+) = \lim_{h\to 0} \frac{2\cos(1+h)\,\pi + \tan^{-1}(1+h) - a - b}{h}$$

$$= \lim_{h\to 0} \frac{(-2\cos\pi h + \tan^{-1}(1+h) - a - b)}{h}$$

Function is differentiable

$\therefore \quad -2 + \dfrac{\pi}{4} = a + b \qquad .....(1)$

$$= \lim_{h\to 0} \frac{-2\cos\pi h + \tan^{-1}(1+h) - 2 + \pi/2}{h}$$

$$= \lim_{h\to 0} 2\pi \sin \pi h + \frac{1}{1+(1+h)^2} = \frac{1}{2}$$

Now $f'(1^-) = f'(1^+)$

$3a = \dfrac{1}{2}$

$a = \dfrac{1}{6} \qquad ....(2)$

by (1) and (2) $b = \dfrac{\pi}{4} - \dfrac{13}{6}$

Q.7 [7]

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} x^2 e^{2(x-1)} = 1$$

f(1) = 1

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} a\ \text{sgn}\,(x+1)\cos 2(x-1) + bx^2$$

= a.1.1+ b

for continuity a + b = 1

LHD (x = 1) is $\lim\limits_{h\to 0}\dfrac{(1-h)^2 e^{-2h}-1}{h}$ =

$$\lim_{h\to 0} 2e^{-2h} + he^{-2h} + \left(\frac{e^{-2h}-1}{h}\right)$$

= 2 + 0 + 2 = 4

RHD (x = 1)

is $\lim\limits_{h\to 0}\dfrac{a\,\text{sgn}(2+h)\cos 2h + b(1+h)^2 - 1}{h}$

$= \lim\limits_{h\to 0}\dfrac{a\cos 2h + b + bh^2 + 2bh - (a+b)}{h}$

$= \lim\limits_{h\to 0} a\left(\dfrac{\cos 2h - 1}{h}\right) + bh + 2b$ = 2b

f(x) is differentiable at x = 1 if 2b = 4

$\Rightarrow$ b = 2 a = –1

Q.8 [0]

As $0 < \{e^x\} < 1$

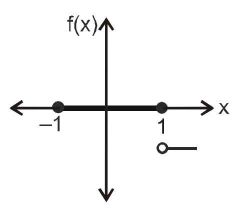
$\therefore \lim\limits_{n\to\infty}\dfrac{\{e^x\}^n - 1}{\{e^x\}^n + 1} = -1$

$\Rightarrow$ $f(x) = -1\ \forall\ x \in R$

Q.9 [3]

f(x) = [x sin πx]

graph of f(x) is as shown in the figure



Q.10 [12]

Given $f''(0) = 4$

$\lim\limits_{x\to 0}\dfrac{2f(x) - 3f(2x) + f(4x)}{x^2}$ $\left[\dfrac{0}{0}\right]$

form

using L′ Hospital rule

$\lim\limits_{x\to 0}\dfrac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x}$ $\left[\dfrac{0}{0}\right]$

form

using L′ Hospital rule

$\lim\limits_{x\to 0}\dfrac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$

$= \dfrac{2.4 - 12.4 + 16.4}{2}$ = 12

Q.11 [11]

f(10 – x) = f(x) = f(4 – x) $\Rightarrow$ f(10 – x) = f(4 – x)

Let 4 – x = t $\Rightarrow$ f(6 + t) = t

$\Rightarrow$ f(x) is periodic with period 6.

$\Rightarrow$ f(x) = 101 at x = 0,6,12, 18, 24, 30

Since f(2 + x) = f(2 – x)

$\Rightarrow$ f(x) is symmetric about x = 2

$\Rightarrow$ f(0) = f(4)

$\Rightarrow$ using periodic nature

f(x) = 101 at x = 4, 10, 16, 22, 28

$\Rightarrow$ f(5 + x) = f(5 – x)

x is symmetric about x = 5

f(0) = f(10) $\Rightarrow$ x = 4, 10, 16, 22

f(6) = f(4) $\Rightarrow$ x = 0, 6, 12, 18,

Total different values of x are 0,4, 6, 10, 12, 16, 18, 22, 24, 28, 30

Q.12 [10] $\sum\limits_{k=1}^{n} f(a+k) = 2048\,(2^n - 1)$

or f(a + 1) + f(a + 2) + ... + f(a + n) = 2048 $(2^n - 1)$

f(x + y) = f(x) . f(y)

f(0) = 1, f(1) = 2

or $f(x) = 2^x$

Now f(a + 1) + f(a + 2) + ... + f(a + n)

$= 2^a\,[2 + 4 + ... + 2^n] = 2^a . 2(2^n - 1)$

or $2048 = 2^{a+1}$ or a = 10

KVPY

PREVIOUS YEAR'S

Q.1 (B)

discontinuous at x = 2

$$f(f(x)) = f\left(\frac{x+5}{x-2}\right)$$

$$\frac{\left(\frac{x+5}{x-2}+5\right)}{\left(\frac{x+5}{x-2}-2\right)} = \frac{6x-5}{-x+9}$$

$$= \frac{6x-5}{9-x}$$

At x = 9 it is discontinuous

Q.2 (B)

$$f(x) = \begin{cases} \sin x, & x \notin Q \\ \tan^2 x, & x \in Q \end{cases}$$

if is continuous at $x = 0, \pi$
so 2 points
$\sin x = \tan^2 x \Rightarrow \sin x(\cos^2 x - \sin x) = 0$
$\sin x = 0$
$x = 0, \pi$
$\sin^2 x + \sin x - 1 = 0$

$$\sin x = \frac{-1 \pm \sqrt{5}}{2}$$

$$\sin x = \frac{\sqrt{5}-1}{2} \quad 2 \text{ values}$$

Total 4 points

Q.3 **(D)**
Definition can be break as

$$g(x) = \int_0^1 f(x-y)dy$$

$x - y = t; -dy. dt$

$$g(x) = \int_{x-1}^{x} f(t)dt$$

$$g(x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x < 1 \\ 2-x & 1 \le x \le 2 \\ 0 & x > 2 \end{cases}$$

Now, check yourself

Q.4 (B)
f (x) = x |sin x| =

$$\begin{Bmatrix} x\sin x, \sin \ge \text{i.e} \in [2n\pi, 2n\pi, 2\pi] \\ -\sin x, \sin x < 0 \text{i} \in (2n\pi + \pi, 24n\pi) \end{Bmatrix}$$

$$f'(n\pi) = \lim_{x\to 4\pi} \frac{f(x) - f(n\pi)}{x - n\pi} = \lim_{x\to 4\pi} \frac{x|\sin x|}{x - 4\pi}$$

$\Rightarrow$ dist at x = 0

Q.5 (C)

$$f(x) = \begin{Bmatrix} x^2 \left|\cos\frac{\pi}{x}\right| & x \ne 0 \\ 0 & x = 0 \end{Bmatrix}$$

The possible of non differntial of f (x)are

$$X = 0, \frac{2}{2n+1} \text{ where } n \in I$$

When x = 0 f (0)= and $\lim_{x\to 0} x^2 \left|\cos\frac{\pi}{x}\right| = 0$

Hence f (x) is continuous at x = 0

Now f (0) = $\lim_{x\to 0} \frac{f(0-h)-0}{-h}$

$$= \lim_{x\to 0} \frac{h^2 \cos\frac{\pi}{h}}{-h} = 0$$

Similarly Rf' (0) = 0 hence differentiable at x = 0

Clearly non differentiable at x = $\frac{2}{2n+1}; n \in I$

Q.6 (A)
f(x) = 1 for x = 0
for $x \ne 0$, f(x) = 1

$\Rightarrow x\sin\frac{1}{x} = 1 \qquad \Rightarrow \sin\frac{1}{x} = \frac{1}{x}$

$\Rightarrow \sin\theta = \theta$ which is true only when $\theta = 0$
As $\theta \ne 0$ so it is not possible

Q.7 **(D)**
Let $f(x) = f(y)$
So, $|f(x) - f(y)| \ge |x - y|$
$\Rightarrow 0 \ge |x - y| \Rightarrow x - y = 0 \Rightarrow x = y$
$\Rightarrow f$ is one-one
Since, f is continuous
So $f(0)$ is finite
Now, $|f(x) - f(0)| \ge |x - 0|$

$\Rightarrow \lim_{x\to\infty} |f(x) - f(0)| \ge \lim_{x\to\infty} |x|$

$\Rightarrow \lim_{x\to\infty} f(x) = \infty$

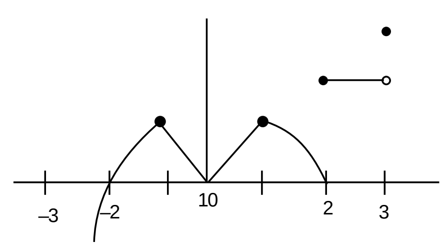
$\Rightarrow f$ is unbounded
$\Rightarrow f$ is surjective

JEE-MAIN PREVIOUS YEAR'S

Q.1 (1)

Non differentiable at x = $-\frac{1}{2}$, 1

Q.2 [5]
Using graph of f(x)



5 point

Q.3 (2)

$$\left|\frac{f(x) - f(y)}{x - y}\right| \le |x - y|$$

$\Rightarrow |f'(x)| \le 0$
$\Rightarrow f'(x) = 0$

$\Rightarrow$ f(x) = constant

$\Rightarrow$ f(x) = 1

Q.4 (1)

If f is continuous at x=–1, then

$f(-1^-)=f(-1)$

$\Rightarrow 2=|a-1+b|$

$\Rightarrow |a+b-1|=2$ ....(i)

similarly

$f(1^-)=f(1)$

$\Rightarrow |a+b+1|=0$

$\Rightarrow a+b=-1$

Q.5 (1)

Doubtful points are x = n, $n \in I$

$$\text{L.H.L} = \lim_{x\to n^+} [x-1] \cos\left[\frac{2x-1}{2}\right]\pi$$

$$= (n-2)\cos\left[\frac{2x-1}{2}\right]\pi = 0$$

$$\text{R.H.L.} = \lim_{x\to n^+} [x-1] \cos\left[\frac{2x-1}{2}\right]\pi$$

$$= (n-1)\cos\left[\frac{2x-1}{2}\right]\pi = 0$$

f(n) = 0

Hence continuous

Q.6 (2)

$$f(og(x)) = \begin{cases} g(x)+2, & g(x)<0 \\ (g(x))^2, & g(x) \ge 0 \end{cases}$$

$$= \begin{cases} x^3+2, & x<0 \\ x^6, & x\in[0,1) \\ (3x-2)^2, & x\in[1,\infty) \end{cases}$$

$$(f og(x))' = \begin{cases} 3x^2, & x<0 \\ 6x^5, & x \quad (0,1) \\ 2(3x-2)\times 3, & x\in(1,\infty) \end{cases}$$

At 'O'

L.H.L. $\neq$ R.H.L. (Discontinuous)

At '1'

L.H.D. = 6 = R.H.D.

$\Rightarrow$ $f$og(x) is differentiable for $x \in R-\{0\}$

Q.7 (3)

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-}(x)$$

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)\cdot\sin^{-1}(1-x)}{x(1-x)(1+x)}$$

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)}{x\cdot 1\cdot 1}\cdot\frac{\pi}{2}$$

Let 1 . x2 = cos θ

$$\frac{\pi}{2}\lim_{x\to 0^+}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$\frac{\pi}{2}\lim_{\theta\to 0^+}\frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}} = \frac{\pi}{\sqrt{2}}$$

$$\text{Now, } \lim_{x\to 0^-}\frac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}\left(-\sin^{-1}x\right)}{(1+x)(2+x)(-x)}$$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}}{1\cdot 2}\cdot\frac{\sin^{-1}x}{x} = \frac{\pi}{4}$$

$\Rightarrow$ RHL $\neq$ LHL

Function can't be continuous

$\Rightarrow$ No value of α exist

Q.8 [1]

$$g[f(x)] = \begin{cases} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x)\ge 0 \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x+a<0 \ \& \ x<0 \\ |x-1|+1 & |x-1|<0 \ \& \ x\ge 0 \\ (x+a-1)^2+b & x+a\ge 0 \ \& \ x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0 \ \& \ x\ge 0 \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a) \ \& \ x\in(-\infty,0) \\ |x-1|+1 & x\in\phi \\ (x+a-1)^2+b & x\in[-a,\infty) \ \& \ x\in(-\infty,0) \\ (|x-1|-1)^2+b & x\in R \ \& \ x\in[0,\infty) \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a) \\ (x \quad a \quad 1)^2 \quad b & x \quad [\ a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty) \end{cases}$$

g(f(x)) is continuous

at x = .a & at x = 0

1 = b + 1 & $(a-1)^2+b=b$

b = 0 & a = 1

$\Rightarrow$ a + b = 1

Q.9 **[6]**

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4} = f(0)$$

$$\lim_{x\to 0}\frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4} = \frac{1}{K}$$

$$\lim_{x\to 0} 2\left(\frac{\sin x + x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right) = \frac{1}{K}$$

$$2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6}=\frac{1}{K}$$

K = 6

Q.10 (4)

$$(f)\ x = \begin{cases} \frac{1}{|x|}, & |x|\ge 1 \\ ax^2+b, & |x|<1 \end{cases}$$

at x = 1 function must be continuous

So, 1 = a + b ...(1)

differentiability at x = 1

$$\left(-\frac{1}{x^2}\right)_{x=1} = (2ax)_{x=1}$$

$-1 = 2a \qquad a = -\frac{1}{2}$

(1) $\quad b = 1 + \frac{1}{2} = \frac{3}{2}$

Q.11 (4)

$f(x)$ is continuous at x = 0

$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$ ...(1)

$f(0) = b$ ...(2)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\frac{\sin(a+1)x}{2x}+\frac{\sin 2x}{2x}\right)$$

$$=\frac{a+1}{2}+1 \quad ...(3)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}}$$

$$=\lim_{x\to 0^+}\frac{(x+bx^3-x)}{bx^{5/2}\left(\sqrt{x+bx^3}+\sqrt{x}\right)}$$

$$=\lim_{x\to 0^+}\frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+bx^2}+1\right)}=\frac{1}{2} \quad ...(4)$$

Use (2), (3) & (4) in (1)

$\frac{1}{2} = b = \frac{a+1}{2}+1 \Rightarrow b = \frac{1}{2}$, a = –2

$a + b = \frac{-3}{2}$

Q.12	(1)
Q.13	[4]
Q.14	(1)
Q.15	(1)
Q.16	(3)
Q.17	[39]
Q.18	(3)
Q.19	[5]
Q.20	(2)
Q.21	(2)
Q.22	[1]
Q.23	[2]
Q.24	(3)
Q.25	(1)
Q.26	(1)
Q.27	[14]

JEE -ADVANCED PREVIOUS YEAR'S

Q.1 (B, C, D) or (B,C)

f(x) = kx

Hence f(x) is continuous & differentiable at $x \in \mathbf{R}$ &

$f'(x) = k$ (constant)

Q.2 (A, B, C, D)

(A) at $x = -\frac{\pi}{2}$ $Lf\left(-\frac{\pi}{2}\right) = 0 = f\left(-\frac{\pi}{2}\right)$

$Rf\left(-\frac{\pi}{2}\right) = 0 \Rightarrow$ continuous

(B) at x = 0 $Rf'(0) = 1$

$Lf'(0) = 0 \Rightarrow$ not differentiable

(C) at x = 1 $Rf'(1) = 1$

$Lf'(1) = 1 \Rightarrow$ differentiable at x = 1

(D) at $x = -\frac{3}{2} > -\frac{\pi}{2}$

$\Rightarrow f(x) = -\cos x \Rightarrow$ differentiable at $x = -\frac{3}{2}$

Q.3 (A)

$$f(x) = \frac{x-b}{bx-1} = \frac{1}{b} + \frac{\left(\frac{1}{b}-b\right)}{(bx-1)}$$

$$f'(x) = \frac{-\left(\frac{1}{b}-b\right)}{(bx-1)^2}\ b,\quad f'(x) < 0\ \forall\ x\ (0, 1)$$

Range of f(x) is (–1, b)

so range ≠ co-domain

so f is not invertible

f^{-1} doesnot exist

No comparison with f^{-1}

Q.4 **(B)**

(I) for derivability at x = 0

L.H.D. = $f'(0^-) = \lim_{h\to 0^+} \dfrac{f(0-h)-f(0)}{-h} = \lim_{h\to 0^+}$

$$\frac{h^2.\left|\cos\left(-\frac{\pi}{h}\right)\right| - 0}{-h} = \lim_{h\to 0^+} -h \,.\, \left|\cos\frac{\pi}{h}\right| = 0$$

RHD $f'(0^+) = \lim_{h\to 0^+} \dfrac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+}$

$$\frac{h^2.\left|\cos\left(\frac{\pi}{h}\right)\right| - 0}{h} = 0$$ So f(x) is derivable at x = 0

(ii) check for derivability at x = 2

RHD = $f'(2^+) = \lim_{h\to 0^+} \dfrac{f(2+h)-f(2)}{h} =$

$$\lim_{h\to 0^+} \frac{(2+h)^2.\left|\cos\left(\frac{\pi}{2+h}\right)\right| - 0}{h} = \lim_{h\to 0^+}$$

$$\frac{(2+h)^2.\cos\left(\frac{\pi}{2+h}\right)}{h}$$

$$= \lim_{h\to 0^+} \frac{(2+h)^2.\sin\left(\frac{\pi}{2} - \frac{\pi}{2+h}\right)}{h}$$

$$= \lim_{h\to 0^+} \frac{(2+h)^2.\sin\left(\frac{\pi h}{2(2+h)}\right)}{\left(\frac{\pi}{2(2+h)}\right)h} . \frac{\pi}{2(2+h)}$$

$$= (2)^2 \,.\, \frac{\pi}{2(2)} = \pi$$

LHD $= \lim_{h\to 0^+} \dfrac{f(2-h)-f(2)}{-h}$

$$= \lim_{h\to 0^+} \frac{(2-h)^2.\left|\cos\left(\frac{\pi}{2-h}\right)\right| - 0}{-h}$$

$$= \lim_{h\to 0^+} \frac{(2-h)^2.\left(-\cos\left(\frac{\pi}{2-h}\right)\right) - 0}{-h} = \lim_{h\to 0^+}$$

$$\frac{(2-h)^2\cos\left(\frac{\pi}{2-h}\right)}{h}$$

$$= \lim_{h\to 0^+} \frac{(2-h)^2.\sin\left(\frac{\pi}{2} - \frac{\pi}{2-h}\right)}{h} =$$

$$\lim_{h\to 0^+} \frac{(2-h)^2.\sin\left(-\frac{\pi h}{2(2-h)}\right)}{\left(-\frac{\pi h}{2(2-h)}\right)} . \frac{-\pi}{2(2-h)} = -\pi$$

So f(x) is not derivable at x = 2

Q.5 **(B,D)**

$$\left.\begin{aligned} f(2n) &= a_n \\ f(2n^+) &= a_n \\ f(2n^-) &= b_n + 1 \end{aligned}\right\} \qquad \begin{aligned} a_n &= b_n + 1 \\ a_n - b_n &= 1 \end{aligned}$$

So B is correct

$$\left.\begin{aligned} f(2n+1) &= a_n \\ f((2n+1)^-) &= a_n \\ f((2n+1)^+) &= b_{n+1} - 1 \end{aligned}\right\} \qquad \begin{aligned} a_n &= b_{n+1} - 1 \\ a_n - b_{n+1} &= -1 \\ a_{n-1} - b_n &= -1 \end{aligned}$$

So D is correct

Q.6 **(A,D)**

Consider

h(x) = f(x) – g(x) Assume a < b

h(a) = λ – g(a) > 0

h(b) = f(b) – λ < 0

else if a > b h(a) < 0 and h(b) > 0.

By intermediate value theorem ⇒ h(c) = 0 .....(1)

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$

$(f(c) - g(c))\,(f(c) + g(c) + 3) = 0$

So there exist a 'c' : f(c) – g(c) from (1).

Hence A is correct.

(D) Similarly $(f(c))^2 = (g(c))^2$

$(f(c) - g(c))\,(f(c) + g(c)) = 0$

⇒ (D) is correct.

B & C are wrong as by counter eg

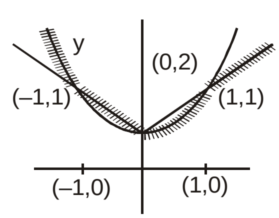
If f(x) = g(x) = λ ≠ 0, then

B → $\lambda^2 + \lambda = \lambda^2 + 3\lambda$ is not possible.

C → $\lambda^2 + 3\lambda = \lambda^2 + \lambda$ is not possible.

Q.7 **[3]**

$$f(x) = |x| + 1 = \begin{cases} x+1 & x \ge 0 \\ -x+1 & x < 0 \end{cases}$$



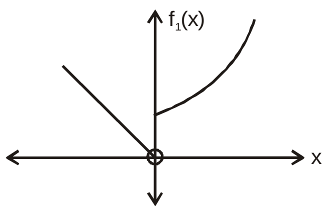
$g(x) = x^2 = 1$

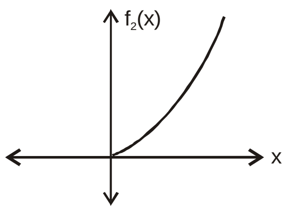
Number of Non-differential points 3.

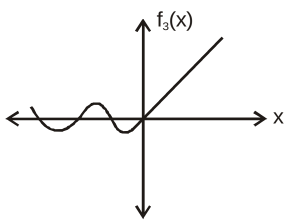
Q.8 **(D)**

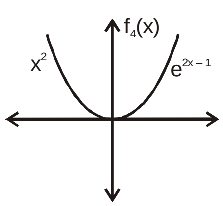
$$f_2(f_1(x)) = (f_1(x))^2 - \begin{cases} x^2 & x < 0 \\ e^{2x} & x \geq 0 \end{cases}$$

$$f_4(x) \begin{cases} x^2 & x < 0 \\ e^{2x} - 1 & x \geq 0 \end{cases}$$









$f_4(x)$ is many-one onto, continuous and non-derivable

$f_3(x)$ is many-one, into, continuous and derivable

$f_2(x)$ is one-one, into, differentiable

Hence $R \to 2$

so (D)

$p \to 1$, $q \to 3$, $R \to 2$, $S \to 4$

Q.9 **(A,D)**

$g(0) = 0$, $g'(0) = 0$ $g'(1) \neq 0$

$$f(x)\begin{cases} g(x) & ; \quad x > 0 \\ -g(x) & ' \quad x < 0 \\ 0 & ; \quad 0 \end{cases} \qquad h(x) = e^{|x|}$$

$$f(h(x)) = g\left(e^{|x|}\right), \quad h(f(x)) = e^{|g(x)|}$$

$$R(f'(0)) = \lim_{x \to 10} \frac{g(x) - g(0)}{x - 0} = g'(0) = 0$$

$$L(f'(0)) = \lim_{x \to 10} \frac{-g(x) - g(0)}{x - 0} = g'(0) = 0$$

$R(h'(0)) = 1$ & $L(h'(0)) = -1$

So h(x) is non derivable. at x = 0

$$\text{Now } \lim_{x \to 0} \frac{f(h(x)) - f(h(0))}{x} = \lim_{x \to 0} \frac{g\left(e^{|x|}\right) - g(1)}{x}$$

$$R(f'(h(x))) = \lim_{x \to 0^+} \frac{g(e^x) - g(1)}{x}$$

$$= \lim_{x \to 0^+} \frac{g(e^x) - g(1)}{e^x - 1} \frac{e^x - 1}{x} = g'(1)$$

$$L(f'(h(x))) = -g'(1)$$

Hence $f(h(x))$ is non derivable at x = 0

Since x = 0 is repeated root of g(x) So $e^{|g|x||}$ is differetiable at x = 0

hence (A), (D)

Q.10 **(A,B,D)**

$f(x) = x\cos(\pi(x + [x])$

Check continuity at x = n

$f(n) = n\cos 2n\pi = n$

$f(n^+) = n\cos 2n\pi = n$

$f(n^-) = n\cos(2n - 1)\pi = -n$

It is discontinous at all integer points except 0

Q.11 [2]

$P(x, y)$: $f(x + y) = f(x)f'(y) + f'(x) f'(y)$ $\forall x, y \in R$

$P(0, 0)$: $f(0) = f(0) f'(0) + f'(0) f(0)$

$\Rightarrow 1 = 2f'(0)$

$$\Rightarrow f'(0) = \frac{1}{2}$$

$P(x, 0)$: $f(x) = f(x) . f'(0) + f'(x).f(0)$

$$\Rightarrow f(x) = \frac{1}{2} f(x) + f'(x)$$

$$\Rightarrow f'(x) = \frac{1}{2} f(x)$$

$$\Rightarrow f(x) = e^{\frac{1}{2}x}$$

$\Rightarrow \text{In}(f(4)) = 2$

Q.12 (D)

(i) $f(x) = \sin\sqrt{1 - e^{-x^2}}$

$$f_1'(x) = \cos\sqrt{1 - e^{-x^2}} \cdot \frac{1}{2\sqrt{1 - e^{-x^2}}} \left(0 - e^{-x^2} \cdot (-2x)\right)$$

at x = 0 $f_1'(x)$ does not exist

So. P → 2

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

$$\lim_{x\to 0^+} \frac{\sin x}{x}\frac{x}{\tan^{-1} x} = 1$$

⇒ $f_2(x)$ does not continuous at x = 0

So Q → 1

(iii) $f_3(x) = \left[\sin \ell n(x+2)\right] = 0$

$$1 < x+2 < e^{\pi/2}$$

$$\Rightarrow 0 < \ell n(x+2) < \frac{\pi}{2}$$

$$\Rightarrow 0 < \sin(\ell n(x+2) < 1$$

$$\Rightarrow f_3(x) = 0$$

So R → 4

(iv) $f_4(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$

So S → 3

Q.13 (C)

f(x) is a non-periodic, continuous and odd function

$$f(x) = \begin{cases} -x^2 + x\sin x, & x < 0 \\ x^2 - x\sin x, & x \geq 0 \end{cases}$$

$$f(-\infty) = \underset{x\to-\infty}{\text{Lt}} (-x^2)\left(1 - \frac{\sin x}{x}\right) = -\infty$$

$$f(-\infty) = \underset{x\to\infty}{\text{Lt}} x^2\left(1 - \frac{\sin x}{x}\right) = \infty$$

⇒ Range of f(x) =R

⇒ f(x) is an onto function .....(1)

$$f'(x) = \begin{cases} -2x + \sin x + x\cos x, & x < 0 \\ 2x - \sin x - x\cos x, & x \geq 0 \end{cases}$$

For (0, ∞)

$f'(x)=(x-\sin x)+x(1-\cos x)$

always +ve or 0   always +ve or 0

⇒ f'(x) > 0

⇒ $f'(x) \geq 0,\ \forall x \in (-\infty, \infty)$

equality at x = 0

⇒ f(x) is one – one function …. (2)

From (1) & (2), f(x) is both one-one & onto.

Q.14 (A,C)

f : R→ R $f(x) = (x^2 + \sin x)(x-1)$

$f(1^+)=f(1^-)=f(1)=0$

fg(x) : f(x).g(x)   fg : R→R

let fg(x) = h(x) = f(x).g(x) h:R→R

option (c) $h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(1) = f'(1)g(1) + 0$,

(as f(1) = 0, g'(x) exists}

⇒ if g(x) is differentiable then h(x) is also differentiable (true) option (A) If g(x) is continuous at

x = 1 then $g(1^+) = g(1^-) = g(1)$

$$h'(1^+) = \lim_{h\to 0^+} \frac{h(1+h) - h(1)}{h}$$

$$h'(1^+) = \lim_{h\to 0^+} \frac{f(1+h)g(1+h) - 0}{h} = f'(1)g(1)$$

$$h'(1^-) = \lim_{h\to 0^+} \frac{f(1-h)g(1-h) - 0}{-h} = f'(1)g(1)$$

So h(x)=f(x).g(x) is differentiable at x=1 (True)

Option (B) (D) $h'(1^+) = \lim_{h\to 0^+} \dfrac{h(1+h) - h(1)}{-h}$

$$h'(1^+) = \lim_{h\to 0^+} \frac{f(1+h)g(1+h)}{h} = f'(1)g(1^+)$$

$$h'(1^-) = \lim_{h\to 0^+} \frac{f(1-h)g(1-h)}{-h} = f'(1).g(1^-)$$

$$\Rightarrow g(1^+) = g(1^-)$$

So we cannot comment on the continuity and differentiability of the function.

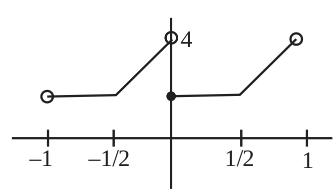
Q.15 [4]

$f(x)=|2x-1|+|2x+1|$

$g(x)=\{x\}$

$f(g(x))=|2\{x\}-1|+|2\{x\}+1|$

$$= \begin{cases} 2 & \{x\} \leq 1/2 \\ 4\{x\} & \{x\} > 1/2 \end{cases}$$



discontinuous at x = 0 ⇒ c = 1

Non differential at $x = -\dfrac{1}{2}, 0, \dfrac{1}{2}$ ⇒ d = 3

∴ c + d = 4

Q.16 (A,B,D)

Since $f(x) = xg(x)$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} xg(x)$$

$$\lim_{x\to 0} f(x) = \left(\lim_{x\to 0} x\right)\cdot\left(\lim_{x\to 0} g(x)\right)$$

$$\lim_{x\to 0} f(x) = 0\times 1 = 0 \qquad \text{....(1)}$$

$f(x+y) = f(x) + f(x) + f(x)\,f(y)$

Now we check continuity of $f(x)$ at $x=a$

$$\lim_{h\to 0} f(a+h) = f(a)+f(b)+f(a)+f(h)$$

$$\lim_{x\to 0} (f(a)+f(h)(1+f(a)))$$

$$\lim_{h\to 0} f(a+h) = f(a)$$

$\therefore f(x)$ is continuous $\forall x \in R$

$$\lim_{x\to 0} f(x) = f(0) = 0 \quad \left(\lim_{x\to 0} f(x) = 0\right)$$

$\therefore f(0) = 0$

and $\lim_{x\to 0} \dfrac{f'(x)}{1} = 1$

$\therefore f'(0) = 1$

Now

$f(x+y) = f(x) + f(y) + f(x)\,f(y)$

using partical derivative (w.r.t. y)

$f'(x+y) + f'(y) + f(x) + f'(y)$

put $y=0$

$f'(x) = f'(0) + f(x)\,f'(0)$

$f'(x) = 1 + f(x)$

$$\int \frac{f'(x)}{1+f(x)}\,dx = \int 1\,dx$$

$$\ell n\,|(1+f(x))| = x + C$$

$f(0)=0$; $c=0$ $\quad \therefore |1+f(x)| = e^x$

$1+f(x) = \pm e^x$ or $f(x) = \pm e^x - 1$

Now $f(0)=0$ $\quad \therefore f(x) = e^x - 1$

$\therefore f(x) = x^e - 1$

option (A) is correct

and $f'(x) = e^x$

$f'(0) = 1$ option (D) is correct

$$g(x) = \frac{f(x)}{x} = \begin{cases} \dfrac{e^x - 1}{x}; & x \neq 0 \\ 1; & x = 0 \end{cases}$$

$$g'(0+h) = \lim_{h\to 0} \frac{g(0+h) - g(0)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{e^h - 1}{h} - 1}{h} = \frac{1}{2}$$

option B is correct

EXERCISES

ELEMENTRY

Q.1 (3)

$$y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$$

Therefore,

$$x^2 . \frac{dy}{dx} - xy + 2 = x^2\left(1 - \frac{1}{x^2}\right) - x\left(1 + \frac{1}{x}\right) + 2 = 0$$

Q.2 (2)

$$\frac{d}{dx}\left(\frac{1}{x^4 \sec x}\right) = \frac{d}{dx}\left(\frac{\cos x}{x^4}\right)$$

$$= \frac{x^4(-\sin x) - \cos x(4x^3)}{(x^4)^2}$$

$$= \frac{-x^3(x\sin x + 4\cos x)}{x^8} = \frac{-(x\sin x + 4\cos x)}{x^5}$$

Q.3 (1)

Here $z = a - \frac{1}{y} \Rightarrow \frac{dz}{dy} = \frac{1}{y^2} = (a - z)^2$

Q.4 (1)

$$\frac{d}{dx}\left(x^2 e^x \sin x\right) = x^2 \frac{d}{dx}\left(e^x \sin x\right) + e^x \sin x \frac{d}{dx}(x^2)$$

$$= xe^x(2\sin x + x\sin x + x\cos x)$$

Q.5 (4)

Since $\frac{dy}{dx} = -\sin(\sin x^2).\cos x^2.2x$

Therefore, at $x = \sqrt{\frac{\pi}{2}}$, $\cos x^2 = \cos\frac{\pi}{2} = 0 \Rightarrow$

$$\frac{dy}{dx} = 0$$

Q.6 (3)

Putting $x = \sin A$ and $\sqrt{x} = \sin B$

$$y = \sin^{-1}(\sin A\sqrt{1 - \sin^2 B} + \sin B\sqrt{1 - \sin^2 A})$$

$$= \sin^{-1}[\sin(A + B)] = A + B = \sin^{-1} x + \sin^{-1}\sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}} + \frac{1}{2\sqrt{x - x^2}}$$

Q.7 (4)

$y = a \sin x + b \cos x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now $\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2$

$$= a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$$

and $y^2 = (a\sin x + b\cos x)^2$

$$= a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$$

So, $\left(\frac{dy}{dx}\right)^2 + y^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$

Hence $\left(\frac{dy}{dx}\right)^2 + y^2 = (a^2 + b^2)$ = constant.

Q.8 (1)

Let $y = \tan^{-1}\sqrt{\frac{1 + \cos\frac{x}{2}}{1 - \cos\frac{x}{2}}} = \tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{4}}{2\sin^2\frac{x}{4}}}$

$$y = \tan^{-1}\cot\frac{x}{4} = \tan^{-1}\tan\left(\frac{\pi}{2} - \frac{x}{4}\right) = \frac{\pi}{2} - \frac{x}{4}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{1}{4}.$$

Q.9 (3)

$$y = \tan^{-1}\frac{4x}{1 + 5x^2} + \tan^{-1}\frac{2 + 3x}{3 - 2x}$$

$$= \tan^{-1}\frac{5x - x}{1 + 5x.x} + \tan^{-1}\frac{\frac{2}{3} + x}{1 - \frac{2}{3}.x}$$

$$= \tan^{-1} 5x - \tan^{-1} x + \tan^{-1}\frac{2}{3} - \tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = \frac{5}{1 + 25x^2}.$$

Q.10 (3)

$$\frac{d}{dx}[\log_7(\log_7 x)] = \frac{d}{dx}\left(\frac{\log_e(\log_7 x)}{\log_e 7}\right)$$

$$= \frac{1}{x\log_e x}\cdot\frac{1}{\log_e 7} = \frac{\log_7 e}{x\log_e x}.$$

Q.11 (1)

$$y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{dy}{dx} = 0, \quad \left\{\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right\}$$

Q.12 (1)

Rationalising,

$$y = \frac{2x^2 + 2\sqrt{x^4-1}}{2} = x^2 + (x^4-1)^{1/2}$$

$$\Rightarrow \frac{dy}{dx} = 2x + \frac{2x^3}{\sqrt{x^4-1}}.$$

Q.13 (1)

$y = (x\cot^3 x)^{3/2}$

$$\therefore \frac{dy}{dx} = \frac{3}{2}(x\cot^3 x)^{1/2}[\cot^3 x + 3x\cot^2 x(-\mathrm{cosec}^2 x)]$$

$$= \frac{3}{2}(x\cot^3 x)^{1/2}[\cot^3 x - 3x\cot^2 x\,\mathrm{cosec}^2 x]$$

Q.14 (2)

$$\frac{d}{dx}[\sqrt{\sec^2 x + \mathrm{cosec}^2 x}] = \frac{d}{dx}\left[\sqrt{\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right)}\right]$$

$$= \frac{d}{dx}[2\,\mathrm{cosec}\,2x] = -4\,\mathrm{cosec}\,2x\cot 2x.$$

Q.15 (3)

$$y = \log x . e^{(\tan x + x^2)}$$

$$\therefore \frac{dy}{dx} = e^{(\tan x + x^2)}.\frac{1}{x} + \log x . e^{(\tan x + x^2)}(\sec^2 x + 2x)$$

$$= e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x + 2x)\log x\right]$$

Q.16 (2)

$$\frac{d}{dx}\left[\frac{2}{\pi}\sin x^\circ\right] = \frac{d}{dx}\left[\frac{2}{\pi}\sin\frac{\pi . x}{180}\right]$$

$$= \frac{2}{\pi}\frac{\pi}{180}\cos\frac{x\pi}{180} = \frac{\cos x^\circ}{90}$$

Q.17 (1)

$$\frac{d}{dx}[\log\sqrt{\sin\sqrt{e^x}}] = \frac{d}{dx}\left[\frac{1}{2}\log(\sin\sqrt{e^x})\right]$$

$$= \frac{1}{2}\cot\sqrt{e^x}\,\frac{1}{2\sqrt{e^x}}e^x = \frac{1}{4}e^{x/2}\cot(e^{x/2})$$

Q.18 (1)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{d}{dt}[a(1-\cos t)]}{\frac{d}{dt}[a(t+\sin t)]}$$

$$\frac{dy}{dx} = \frac{a\sin t}{a + a\cos t} = \frac{\sin t}{1+\cos t} = \frac{2\sin\frac{t}{2}\cos\frac{t}{2}}{2\cos^2\frac{t}{2}}$$

$$\therefore \frac{dy}{dx} = \tan\frac{t}{2}$$

Q.19 (2)

$$x = \frac{2t}{1+t^2},\quad y = \frac{1-t^2}{1+t^2}$$

Put $t = \tan\theta$

$$x = \frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta,\quad y = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2\sin 2\theta}{2\cos 2\theta}$$

$$= -\tan 2\theta = \frac{-2\tan\theta}{1-\tan^2\theta} = \frac{-2t}{1-t^2} = \frac{2t}{t^2-1}.$$

Q.20 (4)

Obviously $x = \cos^{-1}\frac{1}{\sqrt{1+t^2}}$ and $y = \sin^{-1}\frac{t}{\sqrt{1+t^2}}$

$$\Rightarrow \frac{dx}{dt} = -\frac{1}{\sqrt{\frac{t^2}{1+t^2}}}.\frac{(-1)}{2(1+t^2)^{3/2}}2t = \frac{1}{1+t^2}$$

and $\frac{dy}{dt} = \sqrt{1+t^2}.\frac{1}{(\sqrt{1+t^2})^{3/2}} = \frac{1}{1+t^2}$

Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 1$

Q.21 (1)

$y = a\sin^4\theta \Rightarrow \frac{dy}{d\theta} = 4a\sin^3\theta\cos\theta$

and $x = a\cos^4\theta \Rightarrow \frac{dx}{d\theta} = -4a\cos^3\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$

$\therefore \left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1$

Q.22 (3)

Given $y = 3^{x^2}$

$\because \frac{d}{dx}(a^x) = a^x \log_e a$

$\therefore \frac{dy}{dx} = 3^{x^2}\log_e 3\frac{d}{dx}(x^2) \Rightarrow \frac{dy}{dx} = 3^{x^2}.2x.\log_e 3$

Q.23 (1)

Taking log both sides,

$p\log x + q\log y = (p+q)\log(x+y)$

$\Rightarrow \frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = \frac{y}{x}$.

Q.24 (4) $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$

$\Rightarrow y = \sqrt{\sin x + y} \Rightarrow y^2 = \sin x + y$

On differentiating both sides, we get

$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx}(2y-1) = \cos x$.

Q.25 (1)

Given $y = (\sin x)^{\tan x}$; $\log y = \tan x.\log\sin x$

Differentiate with respect to *x*,

$\frac{1}{y}.\frac{dy}{dx} = \tan x.\cot x + \log\sin x.\sec^2 x$

$\frac{dy}{dx} = (\sin x)^{\tan x}[1 + \log\sin x.\sec^2 x]$.

Q.26 (3)

Let $y = \cos^{-1}\sqrt{x}$ and $z = \sqrt{1-x}$

$\therefore \frac{dy}{dz} = \frac{\frac{-1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}}{-\frac{1}{2\sqrt{1-x}}} = \frac{1}{\sqrt{x}}$.

Q.27 (4)

Let $y_1 = \tan^{-1}\sqrt{x}$ and $y_2 = \sqrt{x}$

Differentiating w.r.t. x of y_1 and y_2, we get

$\frac{dy_1}{dx} = \frac{1}{(1+x)}.\frac{1}{2\sqrt{x}}$ and $\frac{dy_2}{dx} = \frac{1}{2\sqrt{x}}$

Hence $\frac{dy_1}{dy_2} = \frac{1}{1+x}$

Q.28 (3)

$y = e^{x^3}$, $z = \log x \Rightarrow \frac{dy}{dx} = e^{x^3}.(3x^2) = 3x^2e^{x^3}$

.....(i)

and $\frac{dz}{dx} = \frac{1}{x}$

....(ii)

$\Rightarrow \frac{dy}{dz} = \frac{3x^2e^{x^3}}{(1/x)} = 3x^3e^{x^3}$

Q.29 (3)

$y = A\cos(nx) + B\sin(nx)$

$\therefore dy/dx = -nA\sin(nx) + nB\cos(nx)$

Again $\frac{d^2y}{dx^2} = -n^2A\cos(nx) - n^2B\sin(nx)$

$= -n^2[A\cos(nx) + B\sin(nx)]$

$\Rightarrow \frac{d^2y}{dx^2} = -n^2y$

Q.30 (1)

$y = e^{\tan^{-1}x} \Rightarrow \frac{dy}{dx} = \frac{e^{\tan^{-1}x}}{1+x^2}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1+x^2).\frac{e^{\tan^{-1}x}}{(1+x^2)} - e^{\tan^{-1}x}(2x)}{(1+x^2)^2}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-2x)e^{\tan^{-1}x}}{(1+x^2)^2}$

$\Rightarrow \frac{d^2y}{dx^2}(1+x^2) = (1-2x)\frac{dy}{dx}$.

Q.31 (3)

$$y = ae^{mx} + be^{-mx};\ \therefore \frac{dy}{dx} = ame^{mx} - mbe^{-mx}$$

Again $\frac{d^2y}{dx^2} = am^2e^{mx} + m^2be^{-mx}$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2(ae^{mx} + be^{-mx}) \Rightarrow \frac{d^2y}{dx^2} = m^2y$$

or $\frac{d^2y}{dx^2} - m^2y = 0$

Q.32 (4)

$$x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x^2(1+y) = y^2(1+x)$$

$$\Rightarrow (x-y)(x+y+xy) = 0$$

$$\Rightarrow x + y + xy = 0,\quad \{\because x \neq y\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

Q.33 (1)

$x^3 + y^3 - 3axy = 0$

Differentiate *w.r.t. x*,

$$3x^2 + 3y^2.\frac{dy}{dx} - 3a\left(x\frac{dy}{dx} + y\right) = 0$$

$$3(x^2 - ay) + 3\frac{dy}{dx}(y^2 - ax) = 0 \Rightarrow \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}.$$

Q.34 (4)

$$\frac{d}{dx}[\tan^{-1}(\cot x) + \cot^{-1}(\tan x)]$$

$$\frac{1(-\mathrm{cosec}^2 x)}{1+\cot^2 x} - \frac{1(\sec^2 x)}{1+\tan^2 x} = -1 - 1 = -2.$$

Q.35 (2)

$$f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right] = 2\tan^{-1}(\log x)$$

$$\Rightarrow f'(x) = 2.\frac{1}{1+(\log x)^2}.\frac{1}{x}. \text{Therefore } f'(e) = \frac{1}{e}$$

Q.36 (4)

$y = \cos^{-1}\sqrt{1-t^2} = \sin^{-1} t$

and $x = \sin^{-1}(3t - 4t^3) = 3\sin^{-1} t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{1}{\sqrt{1-t^2}}\right)}{3\left(\frac{1}{\sqrt{1-t^2}}\right)} \Rightarrow \frac{dy}{dx} = \frac{1}{3}$$

Q.37 (4)

$$f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f(x) = x^3(-6p^3 - 4p^2) - x^2(p^3 - 4p) + 3x^2(p^2 + 6p)$$

$$\Rightarrow f(x) = -6p^3x^3 - 4p^2x^3 - x^2p^3 + 4px^2 + 3p^2x^2 + 18px^2$$

$$\frac{d}{dx}f(x) = -18p^3x^2 - 12p^2x^2 - 2xp^3 + 8px + 6p^2x + 36px$$

$$\frac{d^2}{dx^2}f(x) = -36p^3x - 24p^2x - 2p^3 + 8p + 6p^2 + 36p$$

and $\frac{d^3f(x)}{dx^3} = -36p^3 - 24p^2$ = a constant.

Q.38 (2)

$$D = \begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -p^3\cos px & p^4\sin px & p^5\cos px \\ -p^6\sin px & -p^7\cos px & p^8\sin px \end{vmatrix}$$

$$= p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ -\sin px & -p\cos px & p^2\sin px \end{vmatrix}$$

$$= -p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ \cos px & p\sin px & p^2\cos px \\ \sin px & p\cos px & -p^2\sin px \end{vmatrix} = 0.$$

Q.39 (2)

We have $\frac{dx}{dt} = 1 - \frac{1}{t^2},\ \frac{dy}{dt} = 1 + \frac{1}{t^2}$

$$\therefore \frac{dy}{dx} = \frac{t^2+1}{t^2-1} = \left(1 + \frac{2}{t^2-1}\right) \text{ and}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$$

$= 2.\frac{-1}{(t^2-1)^2}.2t \times \frac{t^2}{t^2-1} = -\frac{4t^3}{(t^2-1)^3}$.

Q.40 (3)

$\frac{dx}{d\theta} = a\cos\theta$ and $\frac{dy}{d\theta} = -b\sin\theta$

$\Rightarrow \frac{dy}{dx} = \frac{-b}{a}\tan\theta$ and $\frac{d^2y}{dx^2} = \frac{-b}{a}\sec^2\theta\frac{d\theta}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a}\sec^2\theta\frac{1}{a\cos\theta} = \frac{-b}{a^2}\sec^3\theta$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 **(1)**

$f'(x) = \sqrt{2x^2-1}$, $y = f(x^2)$

$f'(x^2) = \sqrt{2x^4-1}$, $\frac{dy}{dx} = 2x.f'(x^2)$

$\frac{dy}{dx} = 2x.\sqrt{2x^4-1}$

$\left(\frac{dy}{dx}\right)_{x=1} = 2$

Q.2 **(1)**

$f(x) = \log_x(\ell nx)$

$\Rightarrow f(x) = \frac{\ell n(\ell nx)}{\ell nx}$

$\Rightarrow f'(x) = \frac{\ell nx\left(\frac{1}{\ell nx}.\frac{1}{x}\right) - \frac{1}{x}\ell n(\ell nx)}{(\ell nx)^2}$

$\Rightarrow f'(e) = \frac{\frac{1}{e}-0}{1} = 1/e$

Q.3 **(3)**

$y = x^3 - 8x + 7$ and $x = f(t)$

$\frac{dy}{dt} = 2$ & $x = 3$ at $t = 0$

$\because \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Rightarrow \frac{dx}{dt} = \frac{dy/dt}{dy/dx}$

$\Rightarrow \frac{dx}{dt} = \frac{2}{3x^2-8}$

$\because$ at $t = 0$, $x = 3$

$\therefore \frac{dx}{dt}$ (at $t = 0$) $= \frac{2}{19}$

Q.4 **(2)**

$\sin(xy) + \cos(xy) = 0$

$\Rightarrow \cos(xy)\left(y + x\frac{dy}{dx}\right) - \sin(xy)\left(y + x\frac{dy}{dx}\right) = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{\cos(xy).y - \sin(xy)\cdot y}{\cos(xy).x - \sin(xy).x}$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Q.5 **(3)**

$y = f(x)$

$f(-x) = -f(x) \Rightarrow -f'(-x) = -f'(x)$

$f'(3) = f'(-3) = -2$

Q.6 **(2)**

$y = x - x^2$

$y^2 = x^2 + x^4 - 2x^3$ $\quad u = y^2$

$u = x^2 + x^4 - 2x^3$

$\frac{du}{dx} = 2x + 4x^3 - 6x^2$

$v = x^2 \Rightarrow dv/dx = 2x$

$\frac{du}{dv} = 2x^2 - 3x + 1$

Q.7 **(3)**

$\frac{d}{dx}\left(\frac{x^4+x^2+1}{x^2+x+1}\right) = ax + b \Rightarrow \frac{d}{dx}$

$\left(\frac{(x^2+x+1)\cdot(x^2-x+1)}{(x^2+x+1)}\right) = ax + b$

$\frac{d}{dx}(x^2 - x + 1) = ax + b$

$\Rightarrow 2x - 1 = ax + b$

Q.8 **(3)**

$\left.\frac{dy}{dx}\right|_{x=5} = \frac{x^{5/4}\left(b\frac{3}{2}x^{1/2}\right) - \left(a - bx^{3/2}\right)\frac{5}{4}x^{1/4}}{x^{5/2}} = 0$

$= x^{1/4}\left(xb\frac{3}{2}x^{1/2} - \frac{5}{4}(a + bx^{3/2})\right) = 0$

$= 3\frac{b}{2}x^{3/2} - \frac{5}{4}a - \frac{5}{4}bx^{3/2} = 0$

$= x^{3/2}\left[\frac{3b}{2} - \frac{5b}{4}\right] = \frac{5}{4}a$

Put $x = 4$

$= 5\sqrt{5}\left(\frac{b}{4}\right) = \frac{5}{4}a$

$\boxed{\frac{a}{b} = \sqrt{5}}$

Q.9 **(3)**

$x = \frac{1}{t^3} + \frac{1}{t^2}$

$\frac{dx}{dt} = \frac{-3}{t^4} - \frac{2}{t^3}$, $\frac{dy}{dt} = \frac{3}{2}\left(\frac{-2}{t^3}\right) - \frac{2}{t^2}$

$\frac{dy}{dx} = \frac{-\frac{3}{t^3}-\frac{2}{t^2}}{-\frac{3}{t^4}-\frac{2}{t^3}}$

$\frac{dy}{dx} = t$

so $x\left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = \frac{1+t}{t^3}.t^3 - t = 1$

Q.10 **(3)**

$y = x^{x^2}$

$y = e^{x^2 \ell n x}$

$\frac{dy}{dx} = e^{x^2 \ell n x}.(2x\ \ell n\ x + x)$

$= x^{x^2+1}\ (2\ \ell n\ x + 1)$

Q.11 **(4)**

$f(x) = |x|^{|\sin x|}$

at $x = \pi/4$, $|x| = x$ and $|\sin x| = \sin x$

$\therefore\ f(x) = x^{\sin x}$

$\Rightarrow \ell n(f(x)) = \sin x\ .\ \ell n x$

$\Rightarrow \frac{1}{f(x)}\ f'(x) = \cos x\ \ell n x + \frac{\sin x}{x}$ $\Rightarrow f'(\pi/4) = \left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ell n\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

Q.12 **(1)**

$u = \sec^{-1}\frac{1}{(2x^2-1)}$; $v = \sqrt{1-x^2}$

$u = \cos^{-1}(2x^2-1)$;

differentiating w.r.t. to x

$\frac{du}{dx} = \frac{-1\times(4x)}{\sqrt{1-(2x^2-)^2}}$ & $\frac{du}{dx} = \frac{-x}{\sqrt{1-x^2}}$

$\frac{du}{dv} = \frac{-4x}{\sqrt{-4x^4+4x^2}} \times \frac{\sqrt{1-x^2}}{-x} = \frac{4}{2x}$

$\left|\frac{du}{dv}\right|_{x=1/2} = \frac{4}{2(1/2)} = 4$

Q.13 **(3)**

$x = e^{y+e^{y+.....to\,\infty}}$

$x = e^{y+x}$

$x = e^{(y+x)}\left\{\frac{dy}{dx}+1\right\}$

$\frac{dy}{dx} = \frac{1-e^{x+y}}{e^{x+y}} = \frac{1-x}{x}$

Q.14 **(4)**

$y = \sqrt{\sin x + y}$

squaring both side

$y^2 = \sin x + y$

$2yy' = \cos x + y'$

differentiating w.r.t. to x

$y' = \frac{\cos x}{2y-1}$

Q.15 **(2)**

$y = \cos^{-1}(\cos x)$, $\left.\frac{dy}{dx}\right|_{x=\frac{5\pi}{4}}$

$y' = \frac{-1}{\sqrt{1-\cos^2 x}} \times -\sin x = \frac{\sin x}{|\sin x|}$

$y'|_{x=\frac{5\pi}{4}} = -1$

Q.16 **(3)**

$y = \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$, $|x| > 1$ $\Rightarrow$ y $= \frac{\pi}{2}$

$\frac{dy}{dx} = 0$

Q.17 **(4)**

$y = \sin^{-1}(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}} + p$

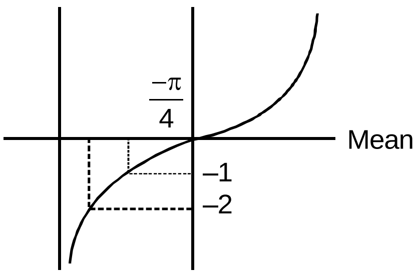
$y = \sin^{-1}(x) + \sin^{-1}(\sqrt{x})$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

Q.18 **(3)**

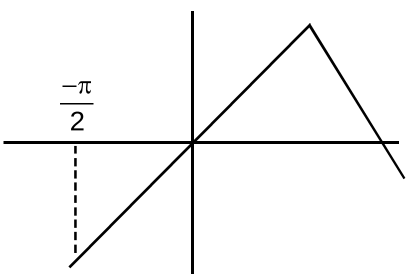
$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \cdot \left.\frac{dy}{dx}\right|_{x=-2}$$

$x = \tan\theta \Rightarrow y = \sin^{-1}(\sin 2\theta)$



$$\theta < -\frac{-\pi}{4}$$

$$2\theta < -\frac{-\pi}{2}$$



$y = \pi - 2\theta = \pi - 2\tan^{-1} x$

$$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-2}{(1+x^2)} = \frac{-2}{5}$$

Q.19 **(2)**

$g(x) = f^{-1}(x)$

$g'(f(x)) \cdot f'(x) = 1$

$$g'(f(x)) = \frac{1}{f'(x)}$$

$$= \frac{1+x^4}{x^5}$$

$$g'(f(g(2))) = \frac{1+a^4}{a^5}$$

Q.20 **(2)**

F'(x) =

$$\begin{vmatrix} f' & g' & h' \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix} + \begin{vmatrix} f & g & h \\ f'' & g'' & h'' \\ f'' & g'' & h'' \end{vmatrix} + \begin{vmatrix} f & g & h \\ g' & g' & h' \\ f''' & g''' & h''' \end{vmatrix} = 0$$

Q.21 **(2)**

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$f'(x)=$

$$\begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$\frac{f'(x)}{x} = \frac{-1}{x}\begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \frac{1}{x}\begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix}$$

$$+\frac{1}{x}\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -\sin x & 1 & 0 \\ 2\dfrac{\sin x}{x} & x & 2 \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 1 & 1 \\ 2\cos x & 2 & 2 \\ \tan x & 1 & 1 \end{vmatrix} +$$

$$\begin{vmatrix} \cos x & x & 1 \\ 2\dfrac{\cos x}{x} & x & 2 \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$\lim_{x\to 0}\frac{f'(x)}{x} = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} + 0 + 0 = 0 - 1(2) + 0 = -2$$

Q.22 **(3)**

$$f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 3x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$$

differentiating w.r.t. to x

$$f'(x) = \begin{vmatrix} \sin x & \cos x & -\sin x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix} +$$

$$\begin{vmatrix} \cos x & \sin x & \cos x \\ -2\sin 2x & 2\cos 2x & -4\sin 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix} +$$

$$\begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ -3\sin 3x & 3\cos 3x & -9\sin 3x \end{vmatrix}$$

$$f'\left(\frac{\pi}{2}\right) = \begin{vmatrix} -1 & 0 & -1 \\ -1 & 0 & -2 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & -1 & 0 \end{vmatrix} +$$

$$\begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 3 & 0 & 9 \end{vmatrix}$$

$= -1(-2) + 0 - 1(1) + 0 - 1(-3) + 0$
$= 2 - 1 + 3 = 4$

Q.23 **(4)**
$x = at^2$
$y = 2at$

$$\frac{dy}{dx} = \frac{2a}{2at}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{t^2}\ \frac{dt}{dx}$$

$$= -\frac{1}{t^2} \cdot \frac{1}{2at} = -\frac{1}{2at^3}$$

Q.24 **(4)**
$y = f(e^x)$
$y' = f'(e^x) \cdot e^x \Rightarrow y'' = f''(e^x)\, e^{2x} + e^x f'(e^x)$

Q.25 **(1)**
$y = e^{-x} \cos x,\ y_4 + ky = 0$
differentiating w.r.t. to x
$y_1 = -e^{-x} \cos x - e^{-x} \sin x$
again differentiating w.r.t. to x
$y_2 = e^{-x} \sin x + e^{-x} \cos x - \{e^{-x} \cos x - e^{-x} \sin x\}$
$y_2 = 2e^{-x} \sin x$
$y_3 = 2e^{-x} \cos x - 2e^{-x} \sin x$
$y_4 = -2e^{-x} \sin x - 2e^{-x} \cos x + 2e^{-x} \sin x$
$- 2e^{-x} \cos x$
$y_4 + 4y = 0$

Q.26 **(4)**
$y = e^{2x}$

$$\frac{dy}{dx} = 2e^{2x} \quad \Bigg| \quad \frac{dx}{dy} = \frac{1}{2}e^{-2x},$$
$$\frac{d^2y}{dx^2} = 4e^{2x} \quad \Bigg| \quad \frac{d}{dy}\left(\frac{dx}{dy}\right) = -e^{-2x} \times \frac{1}{2e^{2x}}$$

$$\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = -\frac{4e^{2x} \times e^{-4x}}{2} = -2e^{-2x}$$

Q.27 **(3)**
$f(x) = x^n$

$$f(1) - \frac{f'(1)}{11} + \frac{f''(1)}{21} - \ldots + (+)\frac{f^n(1)}{n!}$$

$f'(x) = n \cdot x^{n-1} \Rightarrow f'(1) = n$
$f''(x) = n(n-1)\, x^{n-2} \Rightarrow f''(1) = n(n-1)$
$f''(x) = n(n-1)(n-2)\, x^{n-3}$

$\Rightarrow f''(1) = n(n-1)(n-2)$ ⋮

$$f(1-x)^n = 1 - nx + \frac{n(n-1)x^2}{2!} - \ldots\ldots\ldots\ldots$$

$$0 = 1 - n + \frac{n(n-1)}{2!} + \ldots\ldots\ldots\ldots$$

Q.28 **(4)**

$$f'(4) = 5,\ \lim_{x\to 2} \frac{f(4) - f(x^2)}{2 - x} \qquad \left(\frac{0}{0}\right)$$

Apply L. Hospital rule

$$\lim_{x\to 2} 0 - \frac{f'(x^2).2x}{-1} \Rightarrow \lim_{x\to 2} 0 + f'(x^2)\, 2x \Rightarrow f'(4)$$
$\cdot 2 \cdot 2 = f'(4) \cdot 4 = 20$

Q.29 **(2)**
$y = (1 + x)(1 + x^2) \ldots (1 + x^{2n})$

$$y = \frac{(1-x^2)(1+x^2)(1+x^4)\ldots(1+x^{2n})}{(1-x)}$$

$$y = \frac{1-x^{4n}}{1-x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1}) + (1-x^{4n})}{(1-x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-4nx^{4n-1} + 4nx^{4n} + 1 - x^{4n}}{(1-x)^2}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = \frac{-4n \times 0 + 0 + 1 - 0}{1} = 1$$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 **(B)**

$$y = f\left(\frac{2x-1}{x^2+1}\right) \text{ \& } f'(x) = \sin x$$

$$\Rightarrow \frac{dy}{dx} = f'\left(\frac{2x-1}{x^2+1}\right).\left\{\frac{(x^2+1).2-(2x-1).2x}{(x^2+1)^2}\right\} =$$

$$\sin\left(\frac{2x-1}{x^2+1}\right).\frac{2(1+x-x^2)}{(1+x^2)^2}$$

Q.2 **(C)**

$$y = \frac{\tan^{-1}x}{1+\tan^{-1}x}$$

$$\Rightarrow \frac{dy}{d(\tan^{-1}x)} = \frac{1+\tan^{-1}x-\tan^{-1}x}{(1+\tan^{-1}x)^2} =$$

$$\frac{1}{(1+\tan^{-1}x)^2}$$

Q.3 **(B)**

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$x^2(1+y) = y^2(1+x)$

$x^2 - y^2 + x^2y - y^2x = 0$

$(x+y)(x-y) + xy(x-y) = 0$

$(x-y)(x+y+xy) = 0$

$$\because x \neq y \Rightarrow y = -\frac{x}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(1+x)+x}{(1+x)^2} = -\frac{1}{(1+x)^2}$$

Q.4 **(A)**

$$\frac{dy}{dx} = -\left(\frac{2ax+2hy}{2by+2hx}\right)$$

$$= -\frac{y}{x}\left(\frac{ax^2+hy}{by^2+hxy}\right)$$

$$= -\frac{y}{x}\left(\frac{-by^2-2hxy+hxy}{by^2+hxy}\right) = \frac{y}{x}$$

Q.5 **(C)**

$e^{f(x)} = \ell n\, x$

$\because\ f^{-1}(x) = g(x)$

$x = e^{e^{f(x)}}$

$\Rightarrow g(x) = f^{-1}(x) = e^{e^x}$

$\Rightarrow g'(x) = e^{e^x}.e^x = e^{e^x+x}$

Q.6 **(B)**

$$f'(x) = \frac{x^{10}}{1+x^2},\ g(2) = a \text{ \& } g'(2) = ?$$

g is inverse of f

$f(g(x)) = x$

Differentiating w.r.t. x

$f'(g(x)) . g'(x) = 1$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(a)}$$

$$g'(2) = \frac{1+a^2}{a^{10}}$$

Q.7 **(B)**

$$f'(x) = \begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix} +$$

$$\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$= x^2 \sin x + 2x \tan x - 2 \sin x + x^2 \sec^2 x + 2 \sin x - 2x \cos x$

$$\Rightarrow \lim_{x\to 0} \frac{f'(x)}{x} = \lim_{x\to 0}$$

$$\left(x\sin x + 2\tan x - \frac{2\sin x}{x} + x\sec^2 x + \frac{2\sin x}{x} - 2\cos x\right)$$

$= 0 + 0 - 2 + 0 + 0 + 2 - 2 = -2$

Q.8 **(D)**

$$\begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -m^3\cos mx & m^4\sin mx & m^5\cos mx \\ -m^6\sin mx & -m^7\cos mx & m^8\sin mx \end{vmatrix}$$

by expanding = 0

Q.9 **(A)**

$$y = \frac{\pi}{2} - 2\sin^{-1}x.\cos^{-1}x$$

$$\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} - \frac{2\cos^{-1}x}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = 2(\sin^{-1}x - \cos^{-1}x)$$

again differentiate both sides w.r.t x

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 4$$

Q.10 **(C)**

u = ax + b

Let $y = f(ax + b)$

$$\frac{dy}{dx} = a\,f'(ax+b)$$

$$\frac{d^2y}{dx^2} = a^2 f''(ax+b)$$

$$\frac{d^3y}{dx^3} = a^3 f'''(ax+b)$$

$$\vdots$$

$$\frac{d^ny}{dx^n} = a^n f^n(ax+b)$$

$$\Rightarrow \frac{d^n}{dx^n} f(ax+b) = a^n f^n(u) = a^n \frac{d^n}{du^n} f(u)$$

Q.11 **(B)**

$y = x + e^x$

$$\frac{dy}{dx} = 1 + e^x$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{1+e^x}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{-e^x}{(1+e^x)^2}\cdot\frac{dx}{dy} = \frac{-e^x}{(1+e^x)^3}$$

Q.12 **(C)**

$y^2 = P(x)$

$$\Rightarrow 2y\frac{dy}{dx} = P'(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{P'(x)}{2y}$$

$$\Rightarrow 2\frac{d^2y}{dx^2} = \frac{y\,P''(x) - P'(x)\,dy/dx}{y^2}$$

$$\Rightarrow 2y^3\frac{d^2y}{dx^2} = y^2 P''(x) - P'(x)\,.\,y\frac{dy}{dx}$$

$$\Rightarrow 2y^3\frac{d^2y}{dx^2} = P(x)\,P''(x) - \frac{P'(x)P'(x)}{2}$$

$$\because\ y^2 = P(x)\quad \&\quad y\frac{dy}{dx} = \frac{P'(x)}{2}$$

$$\Rightarrow 2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$$

$$= P'(x)\,.\,P''(x) + P(x)\,.\,P'''(x) - 2\,\frac{P'(x).P''(x)}{2}$$

$$\therefore\ 2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right) = P(x)\,.\,P'''(x)$$

Q.13 **(C)**

$y = a\cos \ell n\,x + b\sin \ell n\,x$

differentiating w.r.t. to x

$$y' = -\frac{a}{x}\sin \ell nx + \frac{b}{x}\cos \ell nx$$

$xy' = -a\sin \ell nx + b\cos \ell nx$

$$xy'' + y' = -\frac{a\cos \ell nx}{x} - \frac{b\sin \ell n\,x}{x}$$

$x^2y'' + xy' = -y$

Q.14 **(C)**

$$8\,f(x) + 6\,f\left(\frac{1}{x}\right) = x + 5 \quad .... (i)$$

Replacing x by 1/x

we get $8\,f\left(\frac{1}{x}\right) + 6\,f(x) = \frac{1}{x} + 5$.... (ii)

(i) × 8 ⇒ $64\,f(x) + 48\,f\left(\frac{1}{x}\right) = 8x + 40$.... (iii)

(ii) × 6 ⇒ $36\,f(x) + 48\,f\left(\frac{1}{x}\right) = \frac{6}{x} + 30$..... (iv)

(iii) – (iv) ⇒ $28\,f(x) = 8x - \frac{6}{x} + 10$

Differentiating w.r.t. x

$28\ f'(x) = 8 + \dfrac{6}{x^2}$

Now $y = x^2 f(x)$

$\Rightarrow \dfrac{dy}{dx} = x^2 f'(x) + 2x\ f(x)$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{x=-1} = f'(-1) - 2f(-1)$

$= \dfrac{14}{28} - 2\ (2/7) = \dfrac{-1}{14}$ $\{\because\ f(-1) = 2/7\}$

Q.15 **(B)**

$f(x) = f'(x) + f''(x) + ... \infty$

$f(0) = 1$

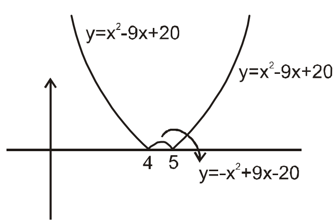
JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 **(B, C, D)**

$f(x) = |(x-4)(x-5)|$

$f'(x) = 2x - 9,\ x > 5$



$f'(x) = -2x + 9,\ 4 < x < 5$

Not defined at x = 4, 5 by graph

Q.2 **(A, C)**

$f_n(x) = e^{f_{n-1}(x)}\ \forall\ n \in N$ and $f_0(x) = x$

$f_1(x) = e^x$

$f_2(x) = e^{e^x}$

$f_3(x) = e^{e^{e^x}}$

:

:

$f_n(x) = e^{e^{e^{\cdots\cdots n \text{ times } x}}} = e^{f_{n-1}(x)}$

Now $\dfrac{d}{dx} f_n(x) = f_n(x)\left(\dfrac{d}{dx} f_{n-1}(x)\right)$

$= f_n(x) .f_{n-1}(x)\left(\dfrac{d}{dx} f_{n-2}(x)\right)$

:

:

$= f_n(x).f_{n-1}(x).f_{n-2}(x)\ ....f_1(x).1$

Q.3 **(A,B,C,D)**

$f'(x) = a \sin x + (ax + b) \cos x + c \cos x - (cx + d) \sin x = x \cos x$

$\Rightarrow (a - cx - d) \sin x + (ax + b + c) \cos x = x \cos x$

$\Rightarrow a \neq 0$ and $a = 1$ and $b + c = 0$ and $a - cx - d = 0$

$\Rightarrow a = 1,\ b = -c$ and $cx = a - d$

$\Rightarrow c = 0$

$\Rightarrow b = c = 0$ and $a = d = 1$

Q.4 **(A, D)**

$x = \cos t,\ y = \ell n\ t$

$\dfrac{dy}{dx} = \dfrac{1}{t}.\dfrac{1}{-\sin t}$

at $t = \dfrac{\pi}{2}$	at $t = \dfrac{\pi}{6}$
$\dfrac{dy}{dx} = \dfrac{-2}{\pi}$	$\dfrac{dy}{dx} = -\dfrac{12}{\pi}$

Q.5 **(A,B,C,D)**

$u = e^x \sin x\ ,\ v = e^x \cos x$

$\dfrac{du}{dx} = e^x \cos x + e^x \sin x$

$\dfrac{dv}{dx} = e^x \cos x - e^x \sin x$

Adding : $\dfrac{du}{dx} + \dfrac{dv}{dx} = 2V = [D]$

$\dfrac{d^2u}{dx^2} = e^x \cos x - e^x \sin x + e^x \cos x + e^x \sin x$

$= 2V \Rightarrow$ [B]

$v\ .\ \dfrac{du}{dx} - u\ .\ \dfrac{du}{dx} = e^{2x} \cos^2 x + e^{2x} \sin x \cos x$

$- e^{2x} \sin x \cos x + e^{2x} \sin^2 x$

$= u^2 + v^2 \rightarrow [A]$

Q.6 **(A,B,D)**

$x^p y^q = (x + y)^{p+q}$

taking log both side

$P\ell nx + q\ell ny = (p + q)\ \ell n\ (x + y)$

$\dfrac{P}{x} + \dfrac{q}{y}\ .\ y' = \dfrac{p+q}{x+y}\ (1 + y')$

$\dfrac{P}{x} + \left(\dfrac{q}{y} - \dfrac{p+q}{x+y}\right) y' = \dfrac{p+q}{x+y}$

$\dfrac{P}{x} + \left\{\dfrac{qx + qy - py - qy}{(x+y)y}\right\} y' = \dfrac{p+q}{x+y}$

$$\left\{\frac{(qx-py)}{(x+y)y}\right\}y' = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\left\{\frac{qx-py}{(x+y)y}\right\}\ y' = \frac{px+qx-px-py}{x(x+y)}$$

$$y' = \frac{(qx-py)}{(qx-py)} \times \frac{(x+y)y}{(x+y)x}$$

$$y' = \frac{y}{x}$$

Q.7 **(A, B)**

$\sin^{-1}\dfrac{t}{\sqrt{1-t^2}}$ w.r.t. $\cos^{-1}\dfrac{t}{\sqrt{1+t^2}}$

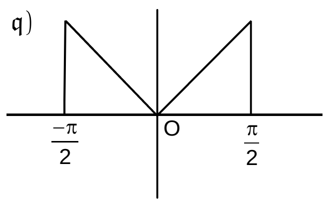
Put $t = \tan\theta \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$$u = \sin^{-1}\left(\frac{\tan\theta}{|\sec\theta|}\right)$$

$u = \sin^{-1}(\sin\theta)$

$u = \theta \quad \tan^{-1} t$

$$\frac{du}{dt} = \frac{1}{(1+t^2)}$$



$$V = \cos^{-1}\frac{1}{|\sec\theta|}$$

$\Rightarrow \cos^{-1}(\cos\theta)$

$\theta \quad \theta > 0$ → $\tan^{-1} > 0$, $t > 0$

$-\theta,\ \theta < 0$ → $t < 0$

$V = \tan^{-1} t$

$$\left.\begin{aligned}\frac{dv}{dt} &= \frac{1}{1+t^2}, \quad t>0\\ \frac{dv}{dt} &= \frac{1}{1+t^2}, \quad t<0\end{aligned}\right] \begin{aligned}&\text{for } t>0,\\ &1\\ &\text{\& for } t<0\\ &-1\end{aligned}$$

Q.8 **(A, B, C, D)**

$2^x + 2^y = 2^{x+y}$ ... (i)

diff. both sides w.r.t.x

$$2^x.\ \ell n2 + 2^y.\ \ell n2\ \frac{dy}{dx} = 2^{x+y}.\ \ell n2\left(1+\frac{dy}{dx}\right)$$

$$2^x - 2^{x+y} = (2^{x+y} - 2^y)\frac{dy}{dx} \quad \text{.... (ii)}$$

$$\frac{2^x(1-2^y)}{2^y(2^x-1)} = \frac{dy}{dx}$$

from (i) & (ii)

$$2^x - 2^x - 2^y = (2^x + 2^y - 2^y)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2^y}{2^x}$$

$$\frac{dy}{dx} = \frac{2^x - 2^{x+y}}{2^{x+y}-2^y} = \frac{-2^y}{2^y(2^x-1)} = \frac{1}{1-2^x} = 1 - 2^y$$

Q.9 **(B, C)**

$$\because\ t = \frac{1}{2}\ell n(x^2+y^2)$$

$$\Rightarrow \frac{1}{2}\ell n(x^2+y^2) = \sin^{-1}\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

case-I : When $x \ge 0$

$$\Rightarrow \ell n(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{x^2+y^2}(2x+2yy') = \frac{2}{1+\frac{y^2}{x^2}}\left(\frac{xy'-y}{x^2}\right)$$

$\Rightarrow xy' - yy' = x + y$

$$\Rightarrow y' = \frac{x+y}{x-y}$$

Let $y = x\tan\theta\ ;\ \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$$\Rightarrow \sin^{-1}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \sin^{-1}\left(\frac{x\tan\theta}{|x\sec\theta|}\right)$$

$$= \sin^{-1}\left(\frac{x}{|x|}\sin\theta\right) = \begin{cases}\theta = \tan^{-1}\left(\dfrac{y}{x}\right), & x \ge 0\\ -\theta = -\tan^{-1}\left(\dfrac{y}{x}\right), & x < 0\end{cases}$$

case-II : When $x < 0$

$$\ell n(x^2+y^2) = -2\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{(2x+2yy')}{x^2+y^2} = \frac{-2}{1+\frac{y^2}{x^2}}\left(\frac{xy'-y}{x^2}\right)$$

$y'(x+y) = y - x$

$$y' = \frac{y-x}{y+x}$$

Q.10 (B, C)

$$y = \cos^{-1}\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1}{2}+\frac{1}{2\sqrt{1+x^2}}\right)}} \cdot \frac{1}{2\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{1+x^2}}}} \cdot$$

$$\frac{1}{2}\ \frac{2x}{(-2)(1+x^2)^{3/2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\frac{1}{4}-\frac{1}{4(1+x^2)}}} \cdot \frac{x}{4(1+x^2)^{3/2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sqrt{1+x^2}}{\sqrt{1+x^2-1}} \cdot \frac{x}{4\sqrt{1+x^2}(1+x^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2|x|(1+x^2)}$$

when $x < 0$

$$\frac{dy}{dx} = \frac{-1}{2(1+x^2)}$$

when $x > 0$

$$\frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

Alternate :

put $x = \tan\theta$

$$\tan^{-1}x = \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$y = \cos^{-1}\sqrt{\frac{1+|\sec\theta|}{2|\sec\theta|}} = \cos^{-1}\sqrt{\frac{\cos\theta+1}{2}}$$

$y = \cos^{-1}(\cos\theta/2)$

$$y = \begin{cases} -\theta/2 , & -\pi/2 < \theta \le 0 \\ \theta/2 , & 0 < \theta < \pi/2 \end{cases}$$

$$\frac{dy}{dx} = \begin{cases} -\frac{1}{2(1+x^2)} , & x \le 0 \\ \frac{1}{2(1+x^2)} , & x > 0 \end{cases}$$

Q.11 (A, B)

$$y = \tan^{-1}\left\{\frac{\ell n\left(\frac{e}{x^2}\right)}{\ell n(ex^2)}\right\} + \tan^{-1}\left(\frac{3+2\ell n x}{1-6\ell n x}\right)$$

$$y = \tan^{-1}\left(\frac{1-2\ell n x}{1+2\ell n x}\right) + \tan^{-1}\frac{3+2\ell n x}{1-6\ell n x}$$

$$\tan y = \frac{\frac{1-2\ell n x}{1+2\ell n x}+\frac{3+2\ell n x}{1-6\ell n x}}{1-\frac{(1-2\ell n x)(3+2\ell n x)}{(1+2\ell n x)(1-6\ell n x)}}$$

$$= \frac{4+16(\ell n x)^2}{-2-8(\ell n x)^2} \Rightarrow -\frac{(2+8(\ell n x)^2)}{1+4(\ell n x)^2}$$

$\Rightarrow -2$

$$\frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx} = 0$$

Q.12 (B,C,D)

$f(x) = \sin 2x\{\sin(x + x^2).\sin(x - x^2) + \cos(x + x^2)\cos(x - x^2)\} + \sin 2x^2\{\cos(x + x^2)\cos(x - x^2) - \sin(x - x^2)\sin(x + x^2)\}$

$\Rightarrow f(x) = \sin 2x.\cos 2x^2 + \cos 2x.\sin 2x^2$

$\Rightarrow f(x) = \sin(2x + 2x^2)$

$\Rightarrow f'(x) = (2 + 4x)\cos(2x + 2x^2)$

Now

$$f'\left(-\frac{1}{2}\right) = (2-2)\cos\left(-1+\frac{1}{2}\right) = 0$$

$f'(-1) = -2\cos 0 = -2$

$f''(x) = 4\cos(2x + 2x^2) - (2 + 4x)^2\sin(2x + 2x^2)$

$f''(0) = 4 - 0 = 4$

Q.13 (C, D)

$f''(x) = -f(x)$ .... (i)

$f'(x) = g(x)$ .... (ii)

$h'(x) = (f(x))^2 + (g(x))^2$ ... (iii)

$h(0) = 2,\ h(1) = 4$

Differentiating equation (ii) w.r.t. x

$f''(x) = g'(x) = -f(x)$

Differentiating equation (iii) w.r.t. x

$h''(x) = 2f(x) . f'(x) + 2g(x) . g'(x)$

$= 2f(x) . f'(x) - 2f'(x) . f(x) = 0$ $\{\because g'(x) = -f(x)\}$

$\Rightarrow h'(x)$ is constant

$\Rightarrow h(x)$ is linear function

$\because h(0) = 2 \Rightarrow h(x)$ not passing through (0, 0)

Let $y = h(x) = ax + b$

at $x = 0$

$y = 2 = b \Rightarrow y = ax + 2$

at x = 1

$a + 2 = 4$

$a = 2$

$\Rightarrow$ curve is $y = 2x + 2$

Q.14 **(A,B,C)**

$f, g, f(0) = \dfrac{2}{g(0)}$

$f'(0) = 2g'(0) = 4g(0),\ g''(0) = 5f''(0) = g(0) = 3$

(A) $h(x) = \dfrac{f(x)}{g(x)} \Rightarrow h'(x) = \dfrac{f'(g) - g'f}{g^2}$

$= h'(0) = \dfrac{f'(0)\,g(0) - g'(0)\,f(0)}{g^2(0)}$

$= h'(0) = \dfrac{4(g(0))^2 - 2g(0).\dfrac{2}{g(0)}}{9}$

$h'(0) = \dfrac{36-4}{9} = \dfrac{32}{5}$ (A)

$k(x) = f(x).\ g(x).\ \sin x$

$k'(x) = f(x)\,g(x)\sin x + f(x)\,g'(x)\sin x + f'(x)\,g(x)\sin x$

$k'(x) = 2$

$\lim\limits_{x\to 0} \dfrac{g'(x)}{f'(x)} = \lim\limits_{x\to 0} \dfrac{g'(x)}{f'(x)} = \dfrac{1}{2}$

Comprehension # 1

Q.15 **(B)**

Q.16 **(A)**

Q.17 **(C)**

(18 to 20)

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{2x+5y-2}{5x+2y+1} = \frac{-5}{8} \text{ at } (1, 1)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) =$$

$$\frac{(5x+2y+1)(2+5\frac{dy}{dx}) - (2x+5y-2)(5+2\frac{dy}{dx})}{(5x+2y+1)^2}$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{(1,1)} = \frac{111}{256}$$

For question 8

Slope of normal at (1, 1) $= -\dfrac{dx}{dy} = \dfrac{8}{5}$

Equation of normal

$y - 1 = \dfrac{8}{5}(x - 1) \Rightarrow 5y - 5 = 8x - 8$

$\Rightarrow 8x - 5y - 3 = 0$

Comprehension # 2

Q.18 **(B)**

$D^*C = (C)'\ 2C$

$= 0.2\ C$

$= 0$

Q.19 **(C)**

$$D^*\left(\frac{f(x)}{g(x)}\right) = \left(\frac{f(x)}{g(x)}\right)' .2\left(\frac{f(x)}{g(x)}\right)$$

$$= \left(\frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)}\right).\ 2\left[\frac{f(x)}{g(x)}\right] \times \frac{g(x)}{g(x)}$$

$$= \frac{(2f(x)f'(x))g^2(x) - (2g(x)g'(x))f^2(x)}{g^4(x)}$$

$$= \frac{D^*f(x).g^2(x) - D^*g(x).f^2(x)}{g^4(x)}$$

Q.20 **(D)**

(21 to 23)

$D^*(f(x)) = \lim\limits_{h\to 0} \dfrac{(f(x+h) - f(a))}{h}.(f(x + h) - f(a))$

$D^*(f(x)) = f'(x).2f(x)$

Now

$D^*(fg) = (fg)'.2fg$

$= (f'(x).g(x) + g'(x).f(x))2f(x)\ g(x)$

$= (2f(x)f'(x))g^2(x) + (2g(x).g'(x))f^2(x)$

$= D^*f(x).g^2(x) + D^*g(x).f^2(x)$

Q.21 (A) $\to$ (p), (B) $\to$ (q), (C) $\to$ (r), (D) $\to$ (s)

(A) $y = \cos^{-1}(\cos x)$

$$y = \frac{-1}{\sqrt{1-\cos^2 x}}\ .\ (-\sin x) = \frac{\sin x}{|\sin x|}$$

$\therefore$ y' at $x = 5$ is -1

(B) $y = f(x) = \ln \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$

$$\therefore \frac{dy}{dx} = \frac{\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\ .\ \frac{1}{2}$$

$$= \frac{1}{2}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)} = \frac{1}{\sin\left(\frac{\pi}{2}+x\right)} =$$

sec x

(C) $\frac{d}{dx}\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\cdot\frac{d}{dx}\left(\frac{1+x}{1-x}\right)$

$$= \frac{(1-x)^2}{2(1+x^2)}\cdot\frac{2}{(1-x)^2} = \frac{1}{1+x^2}$$

at x = –1

$$\frac{d}{dx}\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{1}{2}$$

(D) $\frac{d}{dx}\frac{\ell n|x|}{x} = \frac{x\cdot\frac{1}{x}-\ell n|x|}{x^2} = \frac{1-\ell n|x|}{x^2}$

at x = – 1

$$\frac{d}{dx}\frac{\ell n|x|}{x} = 1$$

Q.22 **(A)–P,Q ; (B)–P,Q,R,S,T ; (C)–P,S ; (D)–R**

(B) 10

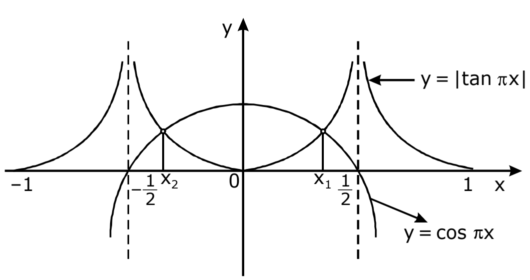
(C) $\frac{4^{x^2}+4^{(x-1)^2}}{2} \ge \left[4^{x^2}.4^{(x-1)^2}\right]$

$\Rightarrow 4^{x^2}+4^{(x-1)^2} \ge \underbrace{2^{x^2+(x-1)^2+1}}_{\min^m \text{ value}=3/2}$

so $\left[4^{x^2}+4^{(x-1)^2}\right]$'s min value = $2^{3/2}$

so $f(x)_{min} = 3/2 = p/q$

(D) Hence at 4 points function is not differentiable



Q.23 (A) → (s); (B) → (r) ; (C) → (s); (D) → (p)

(A) $y = f(x^3)$

$\therefore \frac{dy}{dx} = f'(x^3) . 3x^2$

$\therefore \left.\frac{dy}{dx}\right|_{x=1} = f'(1) . 3 = 9$

(B) f(xy) = f(x) + f(y)

f(1) = f(1) + f(1)

∴ f(1) = 0

∵ $f(1) = f(e) + f\left(\frac{1}{e}\right)$

∴ $f(e) + f\left(\frac{1}{e}\right) = 0$

(C) $f''(x) = -f(x)$, $f'(x) = g(x)$

∴ $g'(x) = f''(x) = -f(x)$

$h(x) = (f(x))^2 + (g(x))^2$

∴ $h'(x) = 2f(x) . f'(x) + 2g(x) . g'(x)$

$= 2f(x) . g(x) + 2g(x)(-f(x)) = 0$

∴ h (x) = c, x ∈ R

∴ h (10) = h (5) = 9.

(D) $y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x)$, $\frac{\pi}{2} < x < \pi$

$$\frac{dy}{dx} = \frac{-\operatorname{cosec}^2 x}{1+\cot^2 x} + \frac{-1}{1+\tan^2 x}.\sec^2 x$$

$= -1 - 1 = -2$

NUMERICAL VALUE BASED

Q.1 [2]

$$\text{LHD} = \lim_{h\to 0}\frac{f(1-h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{a(1-h)^2+1-a-1}{h} = 2a$$

$$\text{RHD} = \lim_{h\to 0}\frac{(1+h)^2+a(1+h)+b-a-1}{h}$$

$$= \lim_{h\to 0} h+2+a+\frac{b}{h} = 2+a$$

Limit exists only when b = 0

$2a = 2+a \Rightarrow a = 2; b = 0 \Rightarrow a+b = 2$

Q.2 [1]

$$f(x) = x + \cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{\ldots}}}$$

$$x+f(x)=2x+\cfrac{1}{2x+\cfrac{1}{2x+..........}}$$

$$x+f(x)=2x+\frac{1}{2x+f(x)}$$

$$(x+f(x))(2x+f(x))=2x[2x+f(x)]+1$$

on differentiation and puting x = 50

$$\frac{f(50)f^1(50)}{50}=1$$

Q.3 [3]

L is $\left(\frac{0}{0}\right)$ from, By L.H. Rule,

$$L=\lim_{h\to 0}\frac{f'(2h+2+h^2)(2+2h)}{f'(h-h^2+1)(1-2h)}$$

$$=\frac{2f'(2)}{f'(1)}=\frac{2\times 6}{4}=3.$$

Q.4 [0]

$$\frac{dy}{dx}=\frac{\sin x}{\sqrt{1-\cos^2 x}}=\frac{\sin x}{|\sin x|}$$

Q.5 [8]

$$y=f[f(f(x))]$$

$$y^1=f^1[f[f(x)]].f^1[f(x)].f^1(x)$$

$y^1(0)=f^1[f[f(0)]].f^1[f(0)].f^1(0)$ = 2.2.2 = 8

Q.6 [1]

$$(x^2+y^2)^2=\left(t-\frac{1}{t}\right)^2=t^2+\frac{1}{t^2}-2$$

$$\Rightarrow x^4+y^4+2x^2y^2=x^2y^4-2 \Rightarrow x^2y^2=-1$$

differenciating with respect to x

$$\Rightarrow x^2.2y\frac{dy}{dx}+y^2.2x=0\Rightarrow x^3y\frac{dy}{dx}=1$$

Q.7 [1]

$$\log(x+y)=2xy;\text{When } x=0; y=1$$

$$\frac{1}{x+y}\left[1+\frac{dy}{dx}\right]=2\left[x\frac{dy}{dx}+y\right]$$

at (0,1)

$$\Rightarrow\frac{dy}{dx}=1$$

Q.8 [2]

Taking log on both sides

n log $(x+y)=\log x+\log y$

differenciating wrt x $\frac{n}{x+y}\left[1+\frac{dy}{dx}\right]=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{y}{x}\left[\frac{x+y-nx}{ny-x-y}\right]\Rightarrow n=2$ satisfies

Q.9 [0]

Given that,

$f(x-y)=f(x).g(y)-f(y).g(x)$ .....(i)

$g(x-y)=g(x).g(y)+f(x).f(y)$ ...(ii)

ln equation (i) putting x = y we get

$$f(0)=f(x)g(x)-f(x)g(x)$$

$$\Rightarrow f(0)=0$$

Putting y = 0 in equation (i) we get

$$f(x)=f(x)g(0)-f(0)g(x)$$

$\Rightarrow f(x)=f(x)g(0)$ [using f(0) = 0]

$$\Rightarrow g(0)=1$$

Putting x = y in equation (ii) we get

$$g(0)=g(x)g(x)+f(x)f(x)$$

$\Rightarrow 1=[g(x)]^2+[f(x)]^2$ [using g(0) = 1)

$\Rightarrow [g(x)]^2=1-[f(x)]^2$ ....(iii)

Clearly g(x) will be differentiable only if $f(x)$ is differentiable.

$\therefore$ First we will check the differentiablity of f(x)

Given that R $f'(0)$ exists

$$i.e., \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} \text{ exists}$$

$$i.e., \lim_{h\to 0}\frac{f(0)g(-h)-f(-h)g(0)}{h} \text{ exists}$$

$$i.e., \lim_{h\to 0}\frac{-f(-h)}{h} \text{ exists}$$

(using $f(0) = 0$ and $g(0) = 1$)

Which can be written as,

$$\lim_{h\to 0}\frac{f(0)-f(-h)}{-h} = Lf'(0)$$

$$\Rightarrow Lf'(0) = Rf'(0)$$

$\therefore$ f is differentiable, at x = 0

Differentiating equation (iii) we get

$$2g(x).g'(x) = -2f(x).f'(x)$$

For x = 0

$$\Rightarrow \quad g(0).g'(0) = -f(0)f'(0)$$

$$\Rightarrow \quad g'(0) = 0$$

[Using $f(0) = 0$ and $g(0) = 1$]

Q.10 [2]

Given that $f(x) = x^3 + e^{x/2}$

Let $g(x) = f^{-1}(x)$ then we should have $gof(x) = x$

$$\Rightarrow \quad g(f(x)) = x$$

$$\Rightarrow \quad g(x^3 + e^{x/2}) = x$$

Differentiating both sides with respect to x, we get

$$g'(x^3 + e^{x/2}).\left(3x^2 + e^{x/2}.\frac{1}{2}\right) = 1$$

$$\Rightarrow \quad g'(x^3 + e^{x/3}) = \frac{1}{3x^2 + e^{x/2}.\frac{1}{2}}$$

For $x = 0$, we get $g'(1) = \frac{1}{1/2} = 2$

Q.11 [0]

We are given the function

$$(\sin y)^{\sin\left(\frac{\pi x}{2}\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan\left[\log(x+2)\right] = 0$$

Differentiating w.r.t. x and put x = - 1 we get $\frac{dy}{dx} = 0$

Q.12 [9]

$$y = \frac{2x^2}{(x-2)(x-3)(x-4)} + \frac{3x + x^2 - 3x}{(x-3)(x-4)} = \frac{x^3}{(x-2)(x-3)(x-4)}$$

$$\log y = 3\log x - \log(x-2) - \log(x-3) - \log(x-4)$$

$$\frac{\frac{dy}{dx}}{y} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x} - \frac{1}{x-3} + \frac{1}{x} - \frac{1}{x-4}$$

$$= \frac{-2}{x(x-2)} - \frac{3}{x(x-3)} - \frac{4}{x(x-4)}$$

$$\frac{x}{y}\frac{dy}{dx} = \frac{2}{2-x} + \frac{3}{3-x} + \frac{4}{4-x} \Rightarrow a + b + c = 9$$

Q.13 [0]

Given that,

$$f(x-y) = f(x).g(y) - f(y).g(x) \text{ .....(i)}$$

$$g(x-y) = g(x).g(y) + f(x).f(y) \text{ ...(ii)}$$

ln equation (i) putting x = y we get

$$f(0) = f(x)g(x) - f(x)g(x)$$

$$\Rightarrow \quad f(0) = 0$$

Q.14 [2]

$$y'(x) = f'\left(f\left(f\left(f(x)\right)\right)\right) f'\left(f\left(f(x)\right)\right) f'\left(f(x)\right) f'(x)$$

$$\Rightarrow y'(0) = f'\left(f\left(f\left(f(0)\right)\right)\right) f'\left(f\left(f(0)\right)\right) f'\left(f(0)\right) f'(0)$$

$$= f'\left(f\left(f(0)\right)\right) f'\left(f(0)\right) f'(0) f'(0)$$

$$= f'\left(f(0)\right) f'(0) f'(0) f'(0)$$

$$= f'(0) f'(0) f'(0) f'(0)$$

$$= \left(f'(0)\right)^4 = 2^4 = 16$$

JEE-MAIN
PREVIOUS YEAR'S

Q. 1 **(1)**

$\ln f(x + 1) = \ln(xf(x))$

$\ln f(x + 1) = \ln x + \ln f(x)$

$\Rightarrow g(x + 1) = \ln x + g(x)$

$\Rightarrow g(x + 1) - g(x) = \ln x$

$$\Rightarrow g''(x + 1) - g''(x) = -\frac{1}{x^2}$$

Put x = 1, 2, 3, 4

$$g''(2) - g''(1) = -\frac{1}{1^2} \quad ...(1)$$

$$g''(3) - g''(2) = -\frac{1}{2^2} \quad ...(2)$$

$$g''(4) - g''(3) = -\frac{1}{3^2} \quad ...(3)$$

$$g''(5) - g''(4) = -\frac{1}{4^2} \quad ...(4)$$

Add all the equation we get

$$g''(5) - g''(1) = -\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2}$$

$$|g''(5) \quad g''(1)| = \frac{205}{144}$$

Q.2 (3)

Q.3 [40]

Q.4 [7]

Q.5 **(3)**

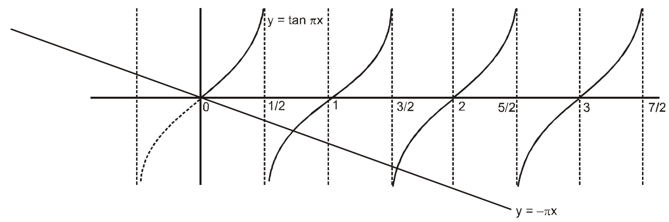
JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 **(B,C)**

$f(x) = x \sin \pi x$, $x > 0$

$f'(x) = \sin \pi x + \pi x \cos \pi x = 0$

$\tan \pi x = -\pi x$



Q.2 **(B, C)** $f : R \to R$

$f(x) = x^3 + 3x + 2$

f is invertible.

Since $g(f(x)) = x$

$\Rightarrow f(x) = g^{-1}(x)$ or $f^{-1}(x) = g(x)$

(A) Since $g(f(x)) = x$

$\Rightarrow g'(f(x)).f'(x) = 1$

For $f(x) = 2$, $x = 0$ so, $g'(2) = \dfrac{1}{f'(0)} \Rightarrow g'(2) = \dfrac{1}{3}$

(B, D) Now,

$$h\left(g\left(g(x)\right)\right) = x \quad \Rightarrow \quad h\left(g\left(g\left(f(x)\right)\right)\right) = f(x)$$

$$\Rightarrow h\left(g(x)\right) = f(x) \quad \Rightarrow \quad h\left(g(3)\right) = f(3) = 38$$

Again,

$$h\left(g(x)\right) = f(x) \quad \Rightarrow \quad h\left(g\left(f(x)\right)\right) = f\left(f(x)\right)$$

$$\Rightarrow h(x) = f\left(f(x)\right) \quad \Rightarrow \quad h'(x) = f'\left(f(x)\right).f'(x)$$

$$\Rightarrow h'(1) = f'\left(f(1)\right).f'(1) = f'(6).f'(1) = 111 \times 6 = 666$$

(C) $g\left(g(x)\right) = 0 \quad \therefore g(x) = g^{-1}(0) \quad \Rightarrow g(x) = f(0) = 2$

$$\Rightarrow x = g^{-1}(2) = f(2) = 16$$

So, $h\left(g\left(g(x)\right)\right) = x \quad \Rightarrow h(0) = 16$

Q.3 (B, C, D)

$$\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$$

$$\lim_{t \to x} \frac{f(x)\cos t - f'(t)\sin x}{1} = \sin^2 x$$

$$\Rightarrow f(x)\cos x - f'(x)\sin x = \sin^2 x$$

$$\Rightarrow -\left(\frac{f'(x)\sin x - f(x)\cos x}{\sin^2 x}\right) = 1$$

$$\Rightarrow -d\left(\frac{f(x)}{\sin x}\right) = 1$$

$$\Rightarrow \frac{f(x)}{\sin x} = x + c$$

Put x = $\frac{\pi}{6}$ & $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$

$\therefore$ c = 0 $\Rightarrow$ f(x) = –xsinx

(A) $f\left(\frac{\pi}{4}\right) = \frac{-\pi}{4}\frac{1}{\sqrt{2}}$

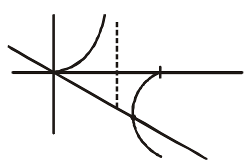
(B) f(x) = –x sinx

as sinx > x–$\frac{x^3}{6}$, $-x\sin < -x^2 + \frac{x^4}{6}$

$$\therefore f(x) < -x^2 + \frac{x^4}{6} \; \forall x \in (0, \pi)$$

(C) $f'(x) = -\sin x - x\cos x$

$f'(x) = 0 \Rightarrow \tan - x \Rightarrow$ there exist $\alpha \in (0, \pi)$ for

which $f'(\alpha) = 0$ 

(D) $f''(x) = -2\cos x + x\sin x$

$$f''\left(\frac{\pi}{2}\right) = \frac{\pi}{2}, f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$$

$$f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$$